

SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

Mathematics in economics

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Differential equations - continued

Now assume that q(x) is not zero:

In such case we use the method called variation of parameters. We assume the solution of the form:

 \mathcal{Y}_{\perp} –

$$\mathcal{Y}_{=}$$

But C is now a function:

$$y_{\pm}$$

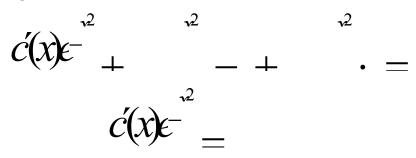
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Differential equations - continued

Substituting the last formula into $\hat{\mathcal{Y}}_{+} =$ yields the solution.

Example: $\mathcal{Y}_{+} \equiv$

Solution: we search for a solution of the form $\mathcal{Y}_{=}$. Substituting into the equation:



Differential equations - continued

Rearranging of terms yields:

Now we integrate:
$$C(x) = \int_{x^2}^{x^2} \frac{x^2}{x^2}$$

Solution of the given equation is:

$$\mathcal{Y}_{\pm}$$

Differential equations – Problem 1

Solve:
$$\dot{\mathcal{Y}}_{-} \frac{\mathbf{I}}{\mathbf{I}} \mathcal{Y}_{-}$$

Solution:

First, we solve corresponding homogenous equation by the separation of variables method:

$$\frac{dy}{dx} = \frac{1}{r} \frac{v}{\sqrt{y}}$$
$$\frac{dy}{y} \frac{dx}{x_{+}}$$
$$\lim_{x \to \infty} \frac{1}{r} \frac{1}{r} \frac{v}{\sqrt{y}}$$

 y_Cx_{\perp}

And finally:

Differential equations – Problem 1 – cont.

In the second step, we apply the variation of a constant method:

$$y_{=}Cx_{-}x_{+}$$

Substitution:

$$C(x)(x_{+}2)_{+}(x) \cdot 1_{-} \frac{1}{2}Q(x) \cdot (x_{+}2)_{=}$$

$$C(x) \cdot (x_{+}2)_{=}$$

$$C(x) \cdot (x_{+}2)_{=}$$

$$C(x)_{=x_{-}}^{x}$$

$$Q(x)_{=x_{-}}^{x}2dx_{-} \frac{x_{+}2}{x_{+}^{2}}dx_{+} \frac{(2)}{x_{-}^{2}}dx_{-}x_{-}2\ln x_{+}2_{+}C$$

$$y_{=}(+2)\kappa_{-}2\ln x_{+}2_{+}$$

The last type of differential equation we will address.

It is of the form: $ay_by_cy_=$

A solution is assumed to be in the form $\mathcal{Y}_{\underline{}}$ where lambda is a root of the so called characteristic equation:

Linear differential equations of the second order with constant coefficients

In the aforementioned three cases, we yield following solutions:

Case 1: $y_{=} z_{+} z_{+} z_{+}^{+}$ Case 2: $y_{=} + z_{+}^{-}$ Case 3: $y_{=} + z_{+}^{-}$

The characteristic equation is a quadratic equation, which means we have three cases:

- Two real roots.
- One real root of the order two.
- Two imaginary roots.

Solve: $\mathcal{Y}_{-} + =$

Solution: we start with the characteristic equation:

x x -

This equation has two real roots: $\lambda_1 = 2$ and $\lambda_2 = 3$.

$$\mathcal{Y}_{=} \mathcal{Y}_{+} \mathcal{Q}_{+} \mathcal$$

Linear differential equations of the second order with constant coefficients - Problem 2

Solve: $\mathcal{Y} _ \mathcal{Y} _ \mathcal{Y} _$

Solution: we start with the characteristic equation:

2 2 -

This equation has two real roots: $\lambda_1 = 3$ and $\lambda_2 = -1$.

$$\mathcal{Y}_{\pm} = \mathcal{Y}_{\pm} - \mathcal{Y}_{\pm} \mathcal{Y}_{\pm}$$

Solve: $\mathcal{Y}_{-} + =$

Solution: we start with the characteristic equation:

λ _ _ _ _

This equation has two real roots: $\lambda_1 = 3$ and $\lambda_2 = 3$.

$$\mathcal{Y}_{\pm}^{-}$$

Solve: $\mathcal{Y}_{+} + =$

Solution: we start with the characteristic equation:

2 - - -

This equation has two real roots: $\lambda_1 = -2+i$ and $\lambda_2 = -2-i$.

$$\mathcal{Y}_{\equiv}$$
 +

Solve: $\mathcal{Y} _ \mathcal{Y} _$

Solution: we start with the characteristic equation:

2 -

This equation has two real roots: $\lambda_1 = 1$ and $\lambda_2 = -1$.

Therefore, the solution is: $\mathcal{Y}_{=} = \mathcal{Y}_{+} \mathcal{Y}_{=}$

Solve: $\mathcal{Y}_{\perp}\mathcal{Y}_{=}$

Solution: we start with the characteristic equation:

2

This equation has two real roots: $\lambda_1 = i$ and $\lambda_2 = -i$.

$$y_{\pm} \operatorname{sin}_{\pm} \cos$$

Linear differential equations of the second order with constant coefficients

Now we will focus on eqautions with non-zero right hand side:

$$ay_{+,+} = x$$

This type of equation is called non-homogenous.

Solution of this equation has the following form:

$$(y_1 + y_2 + x)$$

The solution $G_{\mathcal{Y}_{+}}$ \mathcal{Y}_{2} correspond to a homogenous case, while $H(\mathcal{X})$ is the so called particular integral, which solves a nonhomogenous part of an equation.

A particular integral for the most common functions (polynomials, exponentials, logarithms, etc.) can be easily "guessed".

We will illustrate the procedure by several examples.

Solve: $\dot{\mathcal{Y}}_{-} = = \dot{\mathcal{Y}}_{-}$

Solution: we begin with the homogenous case and its characteristic polynom:

x x -

The roots are $\lambda_1 = 2$ a $\lambda_2 = -1$, hence the solution is:

$$\mathcal{Y}_{\pm}$$
 $\vec{1}^{\mathcal{Z}_{\pm}}$ $\vec{2}^{\xi}$

Now we seek a particular integral in the form:

$$H(x)_{\pm} +$$

Linear differential equations of the second order with constant coefficients – Problem 9 – cont.

Solve: $\mathcal{Y}_{-} = =$

Solution: we substitute y = P(x) into the given equation:

$$- - \mathcal{X}_{+} = 1$$

Which yields: a = -2, b = 2.

Therefore, the general solution to the equation is:

$$y Ce^x Ce^x x_1$$

Solve: $\mathcal{Y}_{+} \equiv -$

Solution: we begin with the homogenous case and its characteristic polynom:

The roots are $\lambda_1 = 0$ a $\lambda_2 = -4$, hence the solution is:

$$y_{\pm}$$
 + ϵ -

Now we seek a particular integral in the form:

Linear differential equations of the second order with constant coefficients – Problem 10 – cont.

Solve: $\dot{\mathcal{Y}}_{-} = = \dot{\mathcal{Y}}_{-}$

Solution: we substitute y = P(x) into the given equation:

$$6ax_{+} + + + = -$$
Which yields: $a_{-} = a_{-} = a_{-}$

Therefore, the general solution to the equation is:

$$G_{+}$$
 $\tilde{\epsilon}_{-}$ $+$ $-$

Problems to solve - 1

Solve:

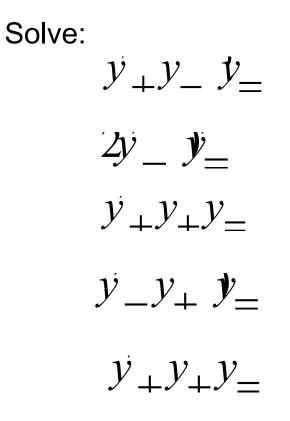
 $y_x_+ y_=$

$$\dot{y}_{-} \dot{y}_{SX} = \hat{y}_{SX}$$

$$y_{+\chi=}^{y}$$

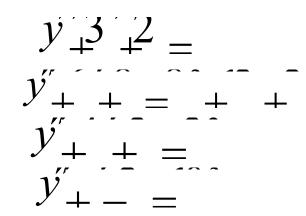
$$\mathcal{Y}_{+}$$
 =

Problems to solve - 2



Problems to solve - 3

Solve:





Final remarks

- See the exam dates in STAG. Everybody has 2 attempts.
- Also, see the older versions of exam tests on my public or Moodle.
- If you need consultations, write me (or Dr. Stoklasova) an email.
- Good luck!

Thank you for your attention!