SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

# Mathematics in Economics

## Lecture 1

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Mathematics in Economics/PMAT

# General information

- Mathematics in Economics: 5 credits, 13 lectures and seminars.
- Seminars are mandatory.
- Materials can be found at Public files and Elearning.
- Teachers: Jiří Mazurek
- Exam: written, two parts (30+70 points).
- Evaluation: A (100-91 points), B (90-81), C (80-71), D (70-66), E (65-60), F (59-0).

# Syllabus (short version)

1. Real function of one real variable
2. Introduction to differential calculus of one real variable
3. Course of a function of one real variable
4. Real function of two real variables
5. Local and bounded extremes of a function of two variables
6. Indefinite integral of one real variable
7. Special substitutions in the indefinite integral
8. Definite integral of one real variable
9. Applications of the definite integral
10. Infinite number series
11. Infinite function series
12. Introduction into ordinary differential equations
13. Linear differential equations

# Literature

*Basic:*

- CHIANG, C.C. *Fundamental Methods of Mathematical Economics*. New York: McGraw-Hill, Inc., 2000. ISBN 0-12-417890-1.

*Recommended*:

- KLEIN, M. Mathematical Methods for Economics. Edinburgh: Pearson Education Limited, 2014.

- ASANO, A. An Introduction to Mathematics for Economics. Tokyo: Sophia University, 2012.

# Youtube math courses

**Bhagwan Singh Vishwakarma:**

https://www.youtube.com/channel/UCBmdoqkBaIUh128ffq8XATw

Nancy Pi:

https://www.youtube.com/channel/UCRGXV1QlxZ8aucmE45tRx8w

# A real function of one real variable

- A **function** is a relation f between two sets X and Y such that each x from X is related to exactly one y from Y.
- We write *y = f(x).*
- **Examples**: $y = \quad +$
  $y =$
- The set **X** is called a **domain**, the set **Y** is called a **range** or a co-domain.
- In economics the domain usually consists of non-negative real numbers.

# A graph of a function

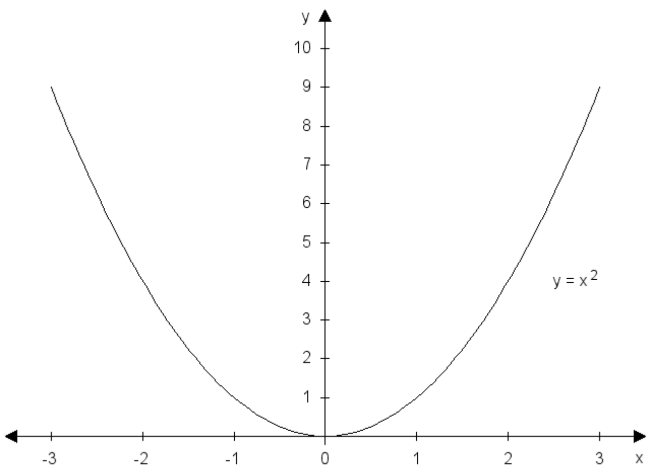
- A **graph** of a function is the collection of all ordered pairs (x,y) displayed in a two-dimensional plane.
- In a two dimensional plane it as a **curve**.

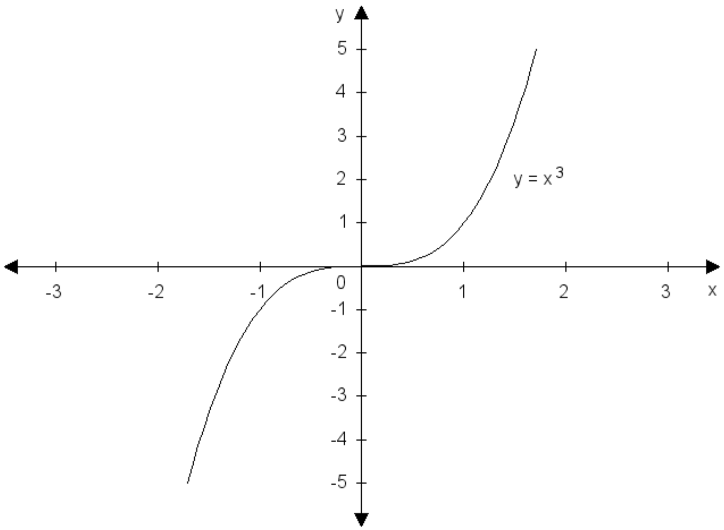
**Examples:**

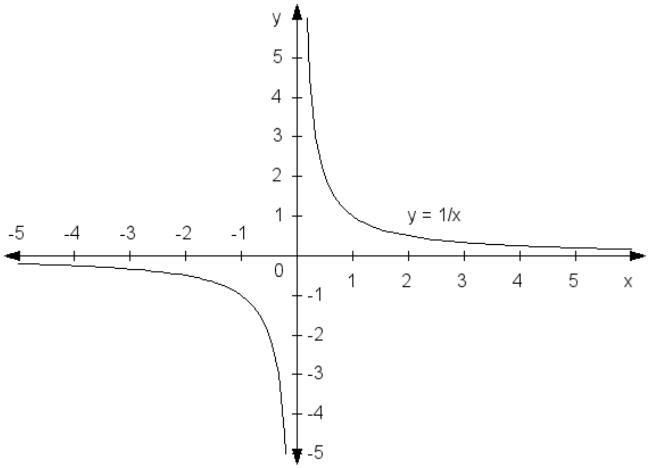
- Linear function: a line,
- Quadratic function: a parabola,
- Reciprocal function: a hyperbola.

Other functions are represented by more complex curves.

# A graph of a function: examples



# Elementary function properties

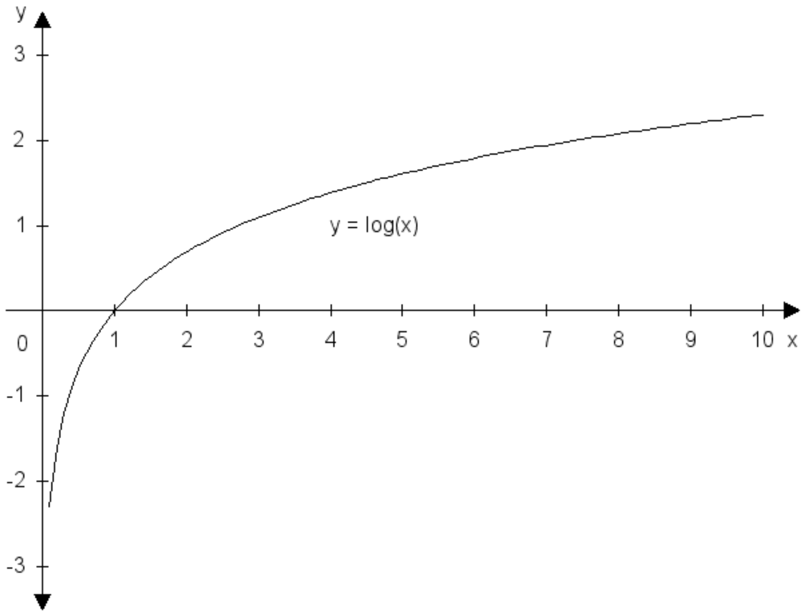
- Domain and range,
- Monotonicity (increasing, decreasing, non-increasing, non-decreasing, constant)
- Extremes (local or global maximum or minimum),
- Concavity and convexity,
- Inflection points,
- Bounded vs unbounded function,
- Even functions and odd functions,
- Peridiocity.

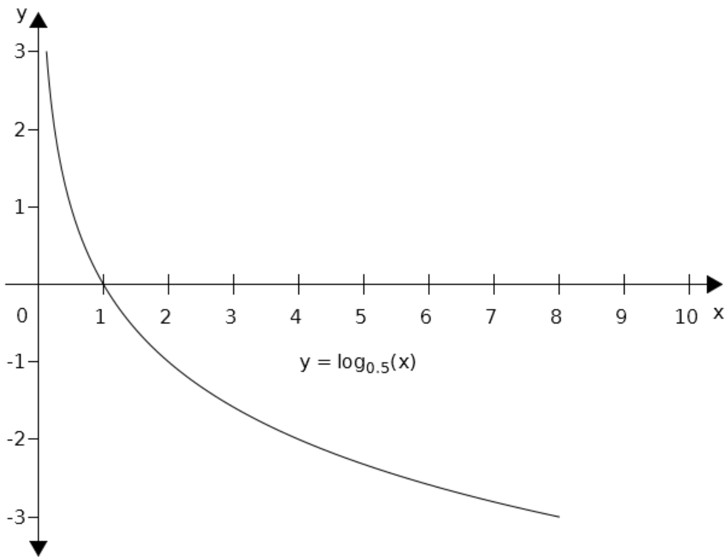
# Elementary functions

- Linear function: $y = ax + b$,
- Quadratic function: $y = \quad + \quad +$
- Polynomial function: $y = \quad + \quad + \quad\quad +$
- Linear reciprocal function: $y =$
- Logarithmic function: $y =$

In the logarithmic function, ***a*** is the so called a ***base***. The **decadic logarithm**: $a = 10$, the **natural logarithm**: $a = e = 2.718\ldots$ For $a$ larger than 1, the function is increasing, and for $a$ smaller than 1 it is decreasing.

# Graphs of logarithmic functions



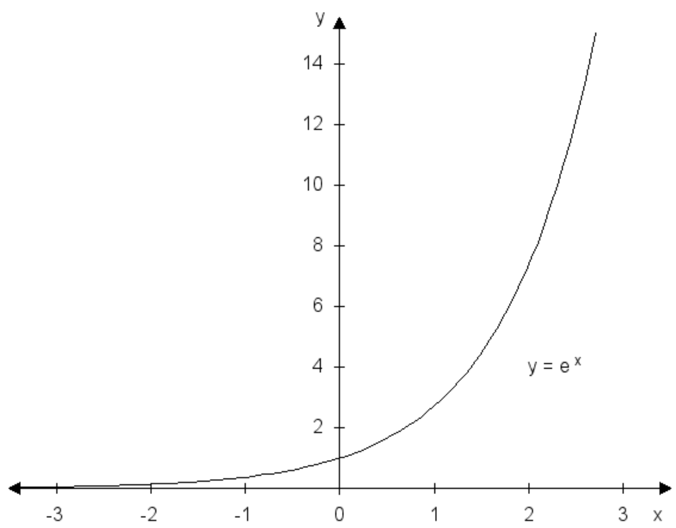



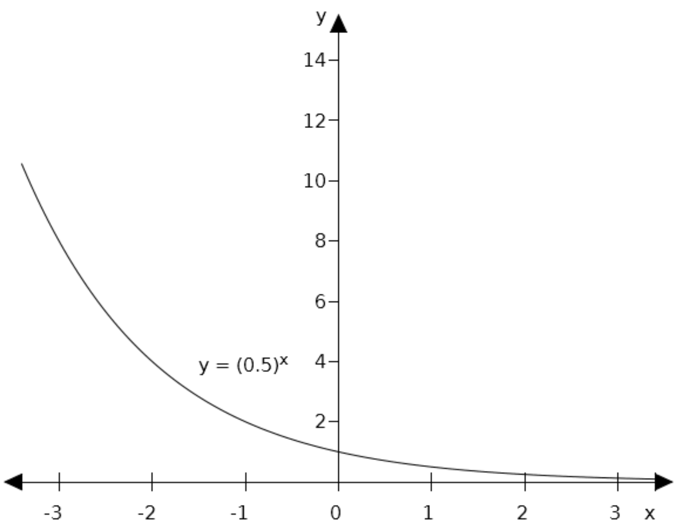
# Elementary functions - continued

- Exponential function: $y = \quad >$
- Goniometric functions: y = sinx, y = cosx, y = tgx, y = cotgx.
- Cyclometric functions: y = arcsinx, y = arccosx, y = arctgx, y = arccotgx.

Notes: exponential and logarithmic functions are inverse to each other. The same applies to goniometric and cyclometric functions.

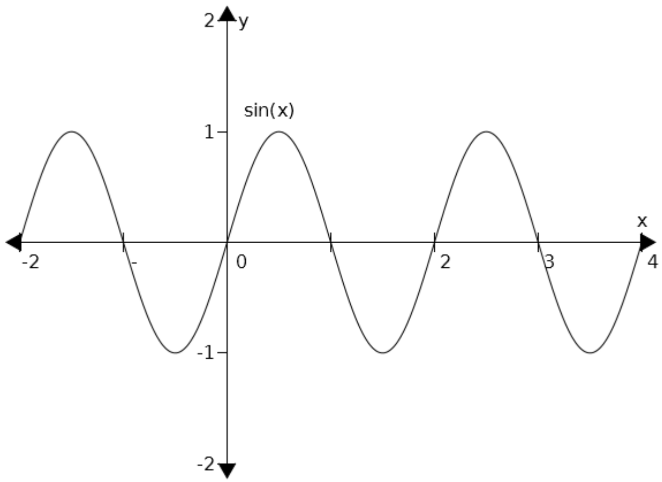
# Graphs of exponential functions

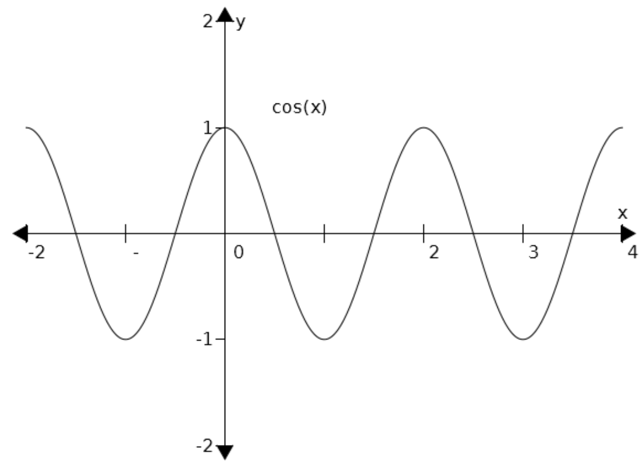


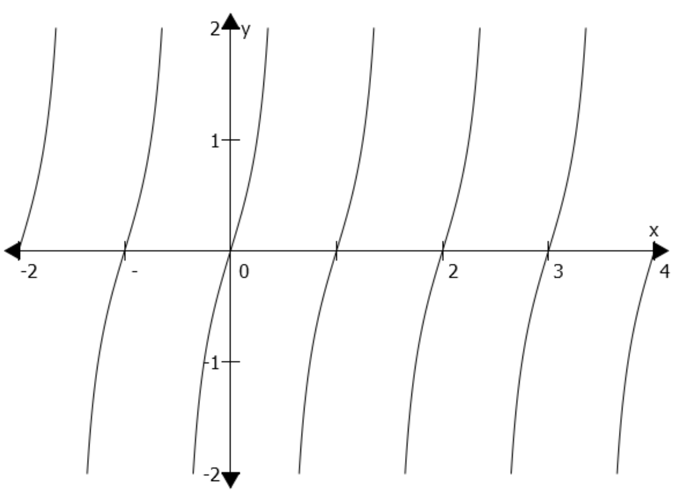



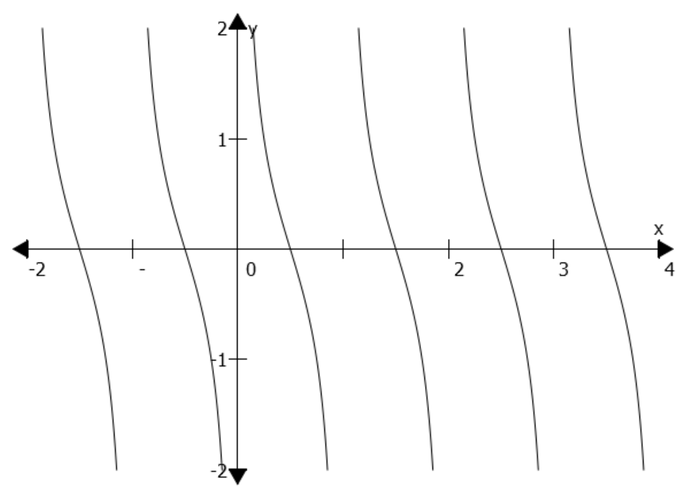

For *a* >1, the exponential function is increasing, for *a* < 1 is decreasing.

# Graphs of goniometric functions



# Polynomials

In economics, various functions, such as demand or supply, are expressed by **polynomials**. Two main tasks when dealing with polynomials are transformation of a polyonomial into a product, and to find roots of a polynomial.

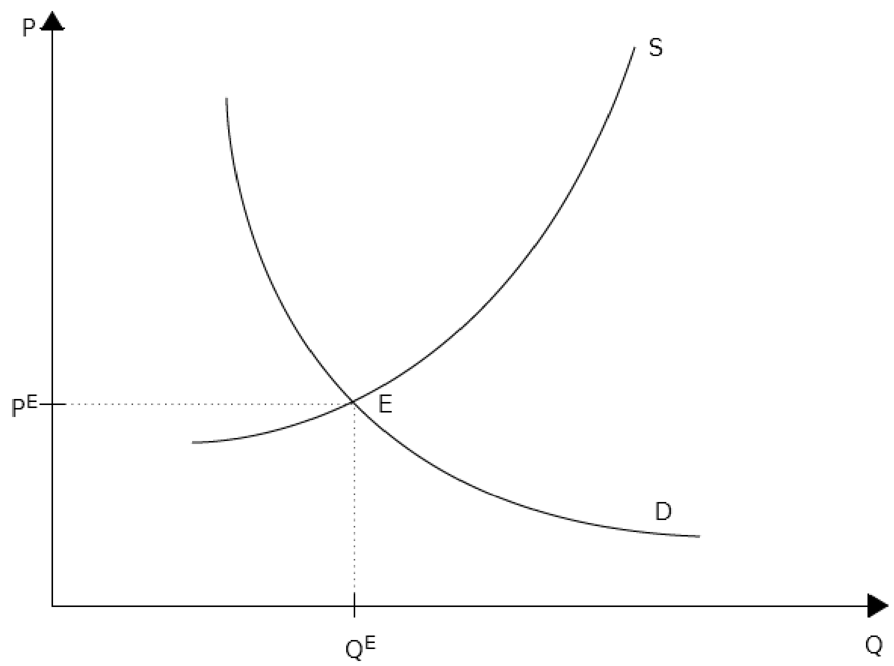
Let $P_n(x)$ denote a polynomial of a degree *n*.

**Polynomial roots** are such values of *x* that $P_n(x) =$ .
The equation above can be solved via known formulas or identities such as $(a + \quad - \quad = \quad -$

# Demand and supply function, equilibrium

- The demand function expresses relationship between a price of a good (P) and a demanded quantity (Q) by customers. Usually, the demand function is denoted as $Q= D(P)$ or $Q_D$ , and it is assumed this function is decreasing.
- The supply function expresses relationship between a price of a good (P) and a supplied quantity (Q) by sellers. Usually, the supply function is denoted as $Q= S(P)$ or $Q_S$ , and it is assumed this function is increasing.
- A point where demand is equal to supply, ant a market is cleared, is called an ***equilibrium***.

# Demand and supply function, equilibrium – cont.



# Solved problem 1

- Find the domain of the function f: $y = \quad - \quad$.

  Solution: the expression under the square root sign must be non-negative, therefore we obtain:

  $$2x - \; >$$
  $$x >$$

- Hence, the domain is $D(f) = \quad \infty$.

# Solved problem 2

- Find the domain of the function f: $y = \quad -$ .

  Solution: the expression in the logarithm must be positive, therefore we obtain:

  $$x^2 - \quad >$$

  We expand the term on the left hand side:

  $$x + \quad - \quad >$$

- From the last inequality it follows that -5 and 5 are the roots that divide the x line into three intervals. By checking the sign in each interval we obtain the final solution:

  $$D(f) = -\infty - \quad \cup \quad \infty$$

# Solved problem 3

- Find the domain of the function f: $y = \qquad -$ .

  Solution: the expression in the arcsin is bounded by -1 from below and by 1 from above. Therefore, we obtain:

  $$- < \; - <$$

  By dividing this inequality into two simple linear inequalities we obtain:

  $2 <$ and $x <$

- Hence, we obtain the solution: $D(f) =$

# Solved problem 4

Let us asssume that the demand and the supply functions are given as follows: $Q_D = \;-\;$ , $Q_S = - +$ .

Find the equilibrium.

Solution: in the equilibrium both functions are equal:

$$IU\_ \quad --\quad \perp$$

Therefore, we obtain: $P_E = 6$, and $Q_E = 4$. Draw both function!

How will the situation change if there is a price floor P = 8?

# Solved problem 5

Let us asssume that the demand and the supply functions are given as follows: $Q_D = \;-\;$ , $Q_S = \;+\;$ .

Find the equilibrium.

Solution: in the equilibrium both functions are equal:

$$24 - \quad - \quad +$$

$$18 -$$

Therefore, we obtain: $P_E = 3$, and $Q_E = 18$. Draw both function!

# Problems to solve

1. Find the domain of the following functions:

$y = \quad -$

$y =$

$y = \quad -$

$y = \quad - \; - \; +$

$y = \quad +$

# Problems to solve – cont.

2. Draw a graph of the following functions:

$$y = \quad +$$

$$y = - \quad -$$

$$y = \quad + \quad +$$

$$y = - \quad + \quad -$$

$$y = \quad +$$

$$y = \quad -$$

$$y = \frac{x}{+}$$

# Problems to solve – cont.

3. For the given functions of demand and supply find the equilibrium both geometrically and algebraically:

$$D(P) = \quad -$$

$$S(P) = \quad +$$

Thank you for your attention!