SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

Mathematics in economics

Lecture 7

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Mathematics in Economics/PMAT

Definite integral

Newton s definite integral:

$$\int^{b} \qquad - $$

In the definition above, F is a primitive function to f,
and *a* and *b* are the limits of the integral.

The result of definite integral is not a function, but a number!

Definite integral – elementary properties

Generally, when computing definite integral, we use the same table of elementary integrals as for an indefinite integral.

The elementary properties of the definite integral:

$$\int^{a}$$

$$\int^{b} \qquad \int^{a}$$

$$\int^{b} \qquad \int^{b}$$

$$\int^{b} \qquad \int^{b} \qquad \int^{b}$$

$$\int^{c} \qquad \int^{b} \qquad \int^{c}$$

Definite integral – a use

The definite integral can be used to calculation of:

- The square under or above given function,
- The length of a curve,
- The volume of a 3D object,
- The area of a 3D object.

Definite integral – An area under/above a function

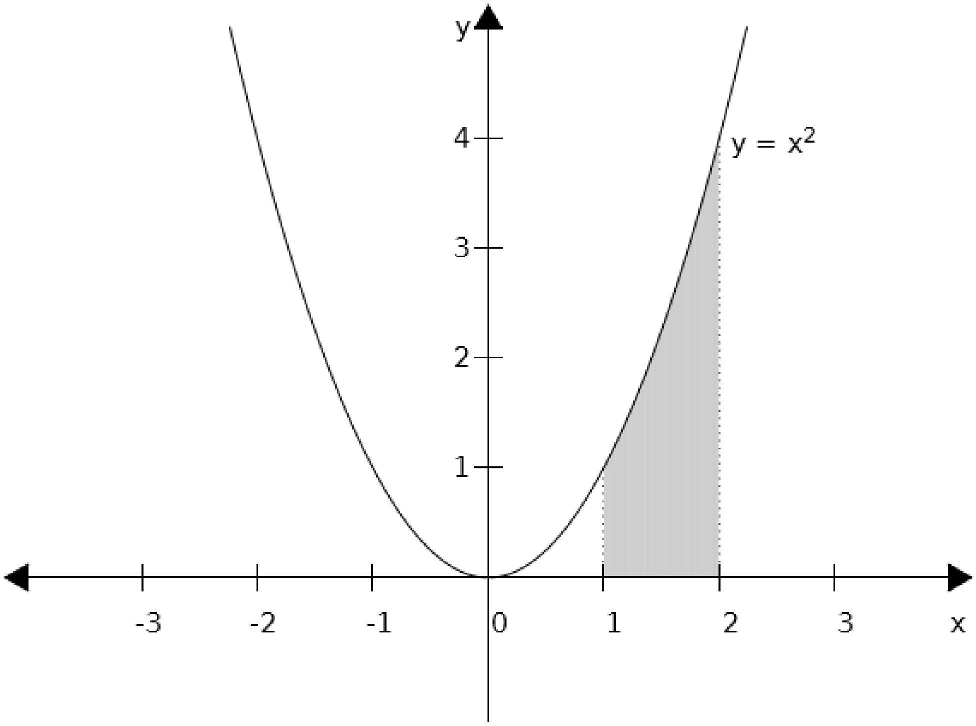
Find: $\int^{2}$

Solution: $\int^{2} \quad \left[\ \right]^{2} \quad 3 \quad 7 \quad =$

What does the number 7/3 mean?

It is the area below the function f(x) on the interval (1,2),
See the next slide for a picture.

Definite integral – An area under/above a function



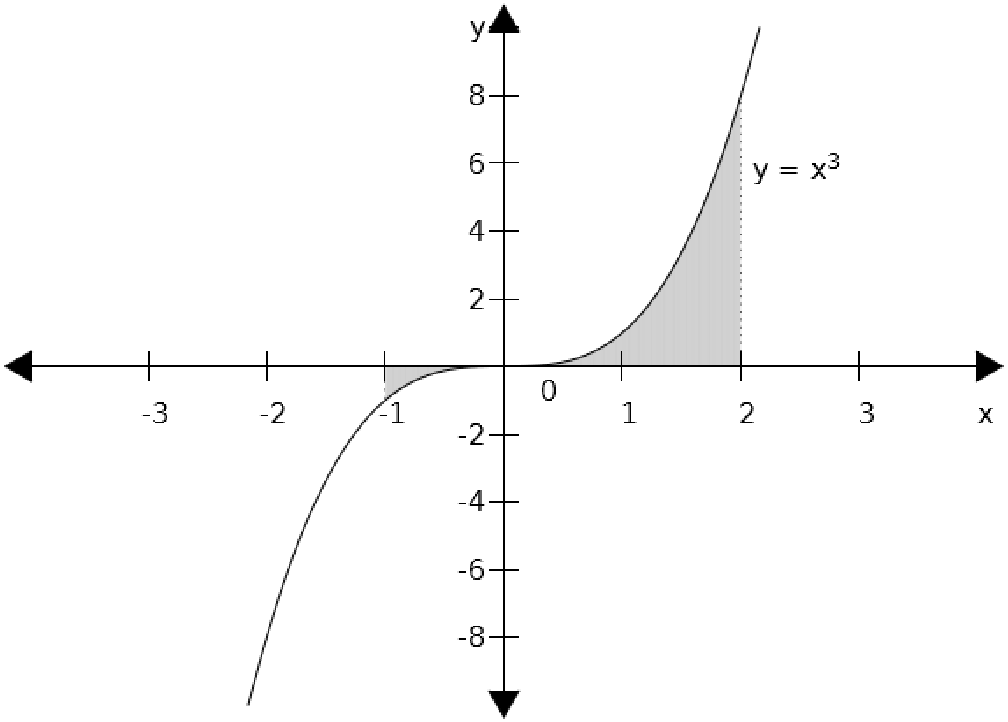
Definite integral – An area under/above a function

Find an area bounded by functions: $y =$, axis x, x = -1 and x = 2.

Solution: We must divide the interval of integration (-1,2) into two intervals: (-1,0) and (0,2) (WHY?):

$$S = \int^{0} \quad \int^{2} \quad \cdots \quad 1 \quad 17 \quad + \; - \; = \; + \; =$$

Definite integral – An area under/above a function



Definite integral – An area under/above a function
Problem 2

Find: $\int^{3}$

Solution: $\int^{3} \; =$

This result means that the area under the function on
the interval (0,3) is 9.

Important note: if a function is positive on the interval of integration, then the result will be a positive number. However, for a negative function the result will be negative!

Definite integral – An area under/above a function
Problems 3 and 4

Find:

Solution:

Find:

Solution:

Definite integral – per partes

Per partes method:

$$\int^{b} \qquad b \quad \int^{b}$$

Example:

$$\int^{2} \qquad \int^{2} \qquad - \quad =$$

Definite integral – substitution

A substitution in an definite integral, example:

$$2x + =$$

$$\int^{3} \qquad 2d \qquad \int^{7} \qquad \int^{17} \qquad 1\left[\;\right]^{7} \qquad \Big(\quad\Big) \qquad =$$

$$= \;\rightarrow\; =$$

Problems to solve - 1

Find: $\int^{4}$

$\int^{4}$

$\int^{3}$

$\int^{e}$

$\int$

An area between two curves

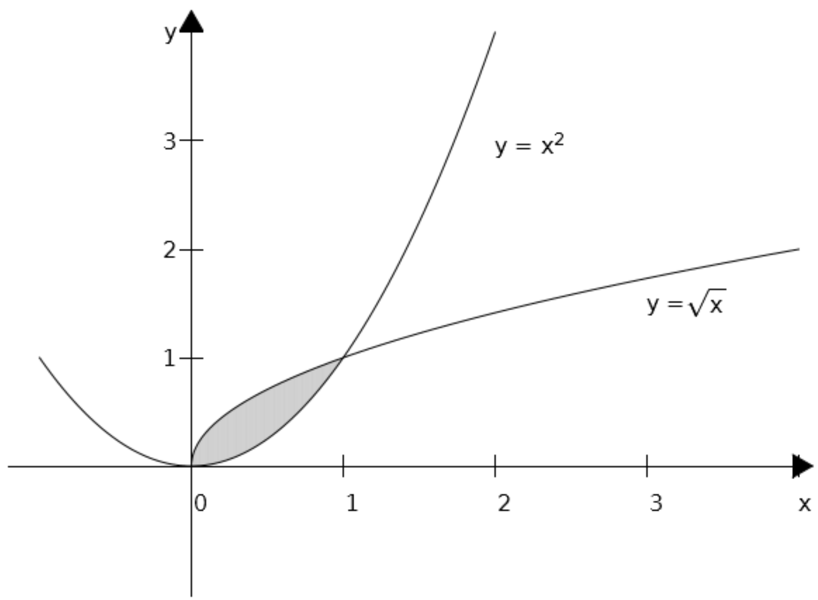
Let f(x) and h(x) be two curves, *S* an area between them
And *a* and *b* their intersections.
Then *S* is given as follows:

$$S = \int^{b}$$

An area between two curves – Problem 1

Find an are between two curves: $y =$ and $y =$.

A picture:



An area between two curves – Problem 1 cont.

Find an are between two curves: $y =$ and $y =$.

Solution:

First, we find intersections: $x^2 =$, hence x = 0 and x = 1.

Now, we can use the integral formula for the area:

$$S = \int^{1} \quad = \Big[\quad \Big]^{1} \Big(\quad \Big) \Big(\quad \Big)$$

An area between two curves – Problem 2

Find an are between two curves: $y=$ and $y=$.

Solution:
First, we find intersections: $x^2 =$, hence x = 0 and x = 2.

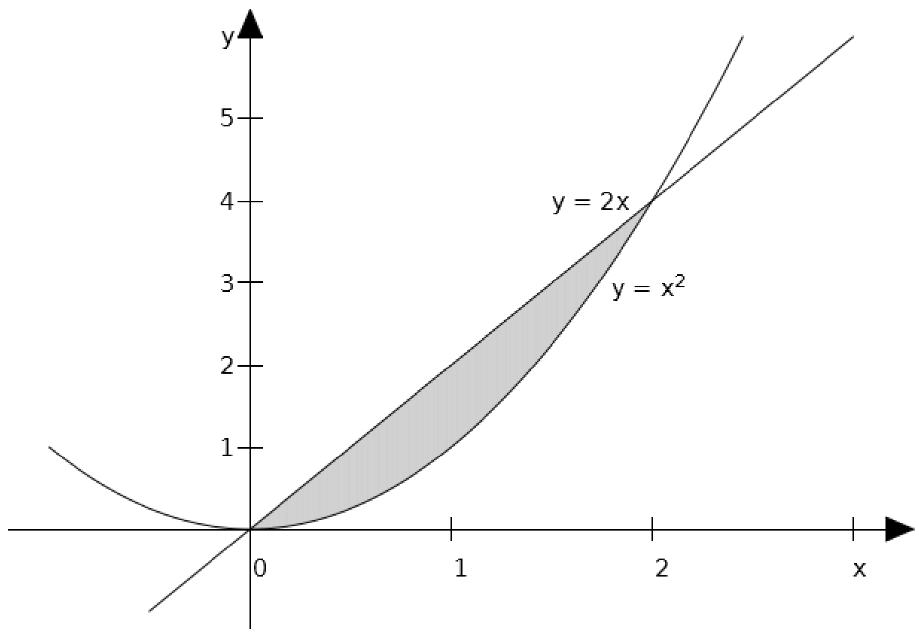
Now, we can use the integral formula for the area:

$$S=\int^{2}$$

An area between two curves – Problem 2 – cont.

Find an are between two curves: $y =$ and $y =$.

A graph:



A volume of a solid of revolution

We assume that a solid is generated by rotating a plane curve around x axis.

In such a case, the volume of a solid is given as:

$$V = \int^{b}$$

A volume of a solid of revolution – Problem 1

Find a volume of a solid generated by a curve $y =$ [illegible], rotating around x axis on the interval (0,3).

Solution:

$$V = \int^{3} (\sqrt{x})^{2}\,dx = \int^{3} x\,dx = \left[x^{2}\right]^{3} = \left(3^{2} - 0^{2}\right) = 9\,\pi$$

Note: the solid is called a rotational parabolloid.

A volume of a solid of revolution – Problem 2

Find a volume of a solid generated by a curve $y =$,
rotating around x axis on the interval (1,2).

Solution:

$$V = \int^{2} (x^2)^2\,dx = \int^{2} x^4\,dx = \left[x^5 \right]^{2} = \left(\quad - \ 1^5 \right) = 31\,\pi$$

Note: the solid is also a rotational parabolloid.

A volume of a solid of revolution – Problem 3

Find a volume of a solid generated by a curve $y =$,
rotating around x axis on the interval (1,4).

Solution:

$$V = \int^{4} \quad \int$$

Note: the solid is called a rotational hyperbolloid.

Problems to solve - 1

Find an are under/above the curve $y =$ on the interval (1,3)

Find an are under/above the curve: $y = \quad \in$

Find an are under/above the curve: $y = \quad + \quad \in$

Find a volume of a solid generated by a curve $y =$,
rotating around x axis on the interval (1,2).

Problems to solve - 2

Find an are under/above the curve: $y = \quad + \quad + \quad \in$

Find an are under/above the curve: $y = - \quad + \quad \in -$

Find an are under/above the curve: $y = \quad - \quad + \quad \in$

Find an are under/above the curve: $y = \quad \in$

Problems to solve - 3

Find an are between two curves: $y = \quad =$

Find an are between two curves: $y = \quad =$

Find an are between two curves: $y = {}^{3} \quad =$

Find a volume of a rotational solid: $y = \quad +$ on interval (0,1).

Thank you for your attention