Risk of investments

Lecture of Corporate Finance

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SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

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Outline of the lecture

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- The risk measurement of an individual investment
- Variance coefficient
- The risk measurement of a portfolio investment
- Correlation coefficient

Risk

Definition: Investment risk can be defined as the probability or likelihood of occurrence of losses relative to the expected return on any particular investment.

Description: Stating simply, it is a measure of the level of uncertainty of achieving the returns as per the expectations of the investor. It is the extent of unexpected results to be realized.

Risk is an important component in assessment of the prospects of an investment. Most investors while making an investment consider less risk as favorable. The lesser the investment risk, more lucrative is the investment. However, the thumb rule is the higher the risk, the better the return.

Business risk refers to the basic viability of a business—the question of whether a company will be able to make sufficient sales and generate sufficient revenues to cover its operational expenses and turn a profit.

Measurement of the risk

Variance

$$\sigma^2 = \sum_{i=1}^{n} P_i (R_i - R)^2$$

where: σ^2 – variance
P_i – probability of particular expected returns
R_i – expected returns
R – average expected return (mean)

Standard deviation

$$\sigma = \sqrt{\sum_{i=1}^{n} P_i (R_i - R)^2}$$

where: σ^2 – variance
P_i – probability of particular expected returns
R_i – expected returns
R – average expected return (mean)

Average expected return (mean)

$$R = \sum_{i=1}^{n} P_i R_i$$

where: R – average expected return (mean)
P_i – probability of particular expected returns
R_i – expected returns

Example: Measuring the risk of one investment

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1. Determine the expected rate of return on each investment, variance and standard deviation according to the data in the table.

Variant no.	Economic growth	Probability to reach returns	Returns of project A (v %)	Returns of project B (v %)
1	Recovery	0,3	28	12
2	Average growth	0,4	20	15
3	Stagnation	0,2	14	14
4	Recession	0,1	9	16

Expected rate of profit

$$\boxed{R = \sum_{i=1}^{n} P_i R_i}$$

- The first thing is to explore Expected rate of profit.

$$R_A = \mathbf{0.3} * 28 + \mathbf{0.4} * 20 + \mathbf{0.2} * 14 + \mathbf{0.1} * 9$$

$$R_B = \mathbf{0.3} * 12 + \mathbf{0.4} * 15 + \mathbf{0.2} * 14 + \mathbf{0.1} * 16$$

Variance and standard deviation

$$\sigma_A^2 = \mathbf{0.3}(28 - \boldsymbol{R_A})^2 + \mathbf{0.4}(20 - \boldsymbol{R_A})^2 + \mathbf{0.2}(14 - \boldsymbol{R_A})^2 + \mathbf{0.1}(9 - \boldsymbol{R_A})^2$$

$$\sigma_B^2 = \mathbf{0.3}(12 - \boldsymbol{R_B})^2 + \mathbf{0.4}(15 - \boldsymbol{R_B})^2 + \mathbf{0.2}(14 - \boldsymbol{R_B})^2 + \mathbf{0.1}(16 - \boldsymbol{R_B})^2$$

- And finally, the risk measurement means a root from variance, which is the standard deviation in %.

Variation coefficient

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2. From the table, select only one investment that is most profitable for a rational investor. The criterion is the value of the coefficient of variation.

Investment	Expected rate of return (%)	Standard deviation (%)
X	17	15,5
Y	10	9
Z	14	13

$$CV_X = \frac{15.5}{17} \qquad CV_Y = \frac{9}{10} \qquad CV_Z = \frac{13}{14}$$

- A lower share of risk on the expected rate of return means more suitable investment for a risk averse investor.

Variation coefficient

$$\boxed{CV = \frac{\sigma}{R}}$$

Which is better ??

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3. Consider Asset A with an expected rate of return of 14.5% and a standard deviation of 2.75% and Asset B with an expected rate of return of 17.4% and a standard deviation of 5.25%. Determine which of them is more suitable for investment, justify your answer and support the calculation.

Variation coefficient

$$\boxed{CV = \frac{\sigma}{R}}$$

$$CV_A = \frac{2.75}{14.5}$$

$$CV_B = \frac{5.25}{17.4}$$

- Lower or higher? ☺

Whole example to decide which investment is better

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4. Determine the expected rate of return of each investment, variance and standard deviation according to the data in the table and select only one of the investments that is most beneficial for a rational investor. The criterion is the value of the coefficient of variation.

Variant no.	Economic growth	Probability to reach returns (%)	Returns of investments (%)		
			A	B	C
1	Recovery	20	25	19	20
2	Average growth	40	21	18	15
3	Stagnation	30	15	14	10
4	Recession	10	9	12	10

Solution:

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$$a)\ R_A = 0.2 * 25 + 0.4 * 21 + 0.3 * 15 + 0.1 * 9$$
$$R_B = 0.2 * 19 + 0.4 * 18 + 0.3 * 14 + 0.1 * 12$$
$$R_C = 0.2 * 20 + 0.4 * 15 + 0.3 * 10 + 0.1 * 10$$

$$b)\ \sigma_A^2 = 0.2(25 - R_A)^2 + 0.4(21 - R_A)^2 + 0.3(15 - R_A)^2 + 0.1(9 - R_A)^2$$
$$\sigma_B^2 = 0.2(19 - R_B)^2 + 0.4(18 - R_B)^2 + 0.3(14 - R_B)^2 + 0.1(12 - R_B)^2$$
$$\sigma_C^2 = 0.2(20 - R_C)^2 + 0.4(15 - R_C)^2 + 0.3(10 - R_C)^2 + 0.1(10 - R_C)^2$$

$$c)\ \sigma_A = \sqrt{\sigma_A^2}$$
$$\sigma_B = \sqrt{\sigma_B^2}$$
$$\sigma_C = \sqrt{\sigma_C^2}$$

$$d)\ CV_A = \frac{\sigma_A}{R_A}$$
$$CV_B = \frac{\sigma_B}{R_B}$$
$$CV_C = \frac{\sigma_C}{R_C}$$

- Always, please, just follow these steps! ☺

Correlation coefficients

$$\sigma^2 = \sum_{i=1}^{n} P_i (R_i - R)^2$$

where: σ^2 – variance
P_i – probability of particular expected returns
R_i – expected returns
R – average expected return (mean)

$$cov(R_A, R_B) = \sum_{i=1}^{n} P_i (R_{iA} - R_A)(R_{iB} - R_B)$$

$$k_{A,B} = \frac{cov(R_A, R_B)}{\sigma_A \sigma_B}$$

where: $k_{A,B}$ – correlation coefficient of asset A and asset B
$cov(R_{A,B})$ – covariance of assets A and B
$\sigma_{A,B}$ – standard deviation of asset A and asset B

Comment:

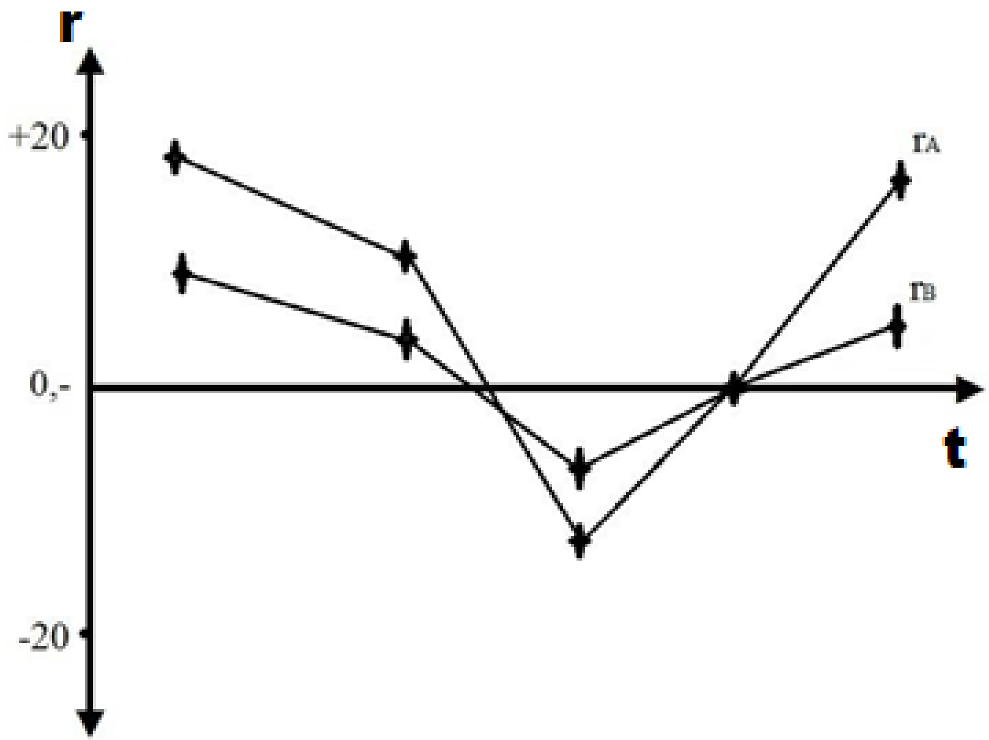
You should be familiar with these formulas how can we compute Parson's correlation coefficients. As the first it is necessary to compute expected return as well as average expected return for asset A and asset B.

Next step is to compute the **variance** to make a root from it, which means standard deviation.

Key role plays a **covariance** when we combining development of returns of both assets. When the covariance is negative, we will compute negative correlation coefficient, as well.

Finally, if we already have computed, standard deviations as well as values of covariance, we are able to compute **correlation coefficients** and compare possible investment pairs.

Positively correlated returns



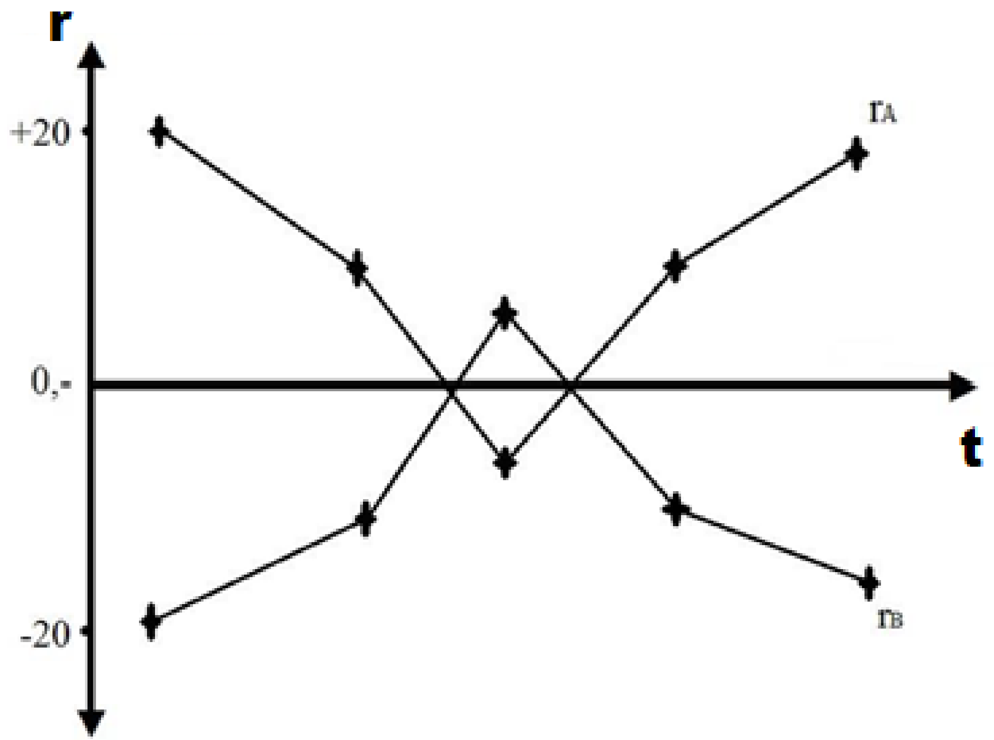
Positively correlated returns

Positively correlated returns

The rate of return on the instruments is identical. When investing in such instruments, the investor does not reduce the risk of his portfolio.

The resulting effect of perfect correlation is practically the same as if it had invested in one asset.

Negatively correlated returns



Negatively correlated returns

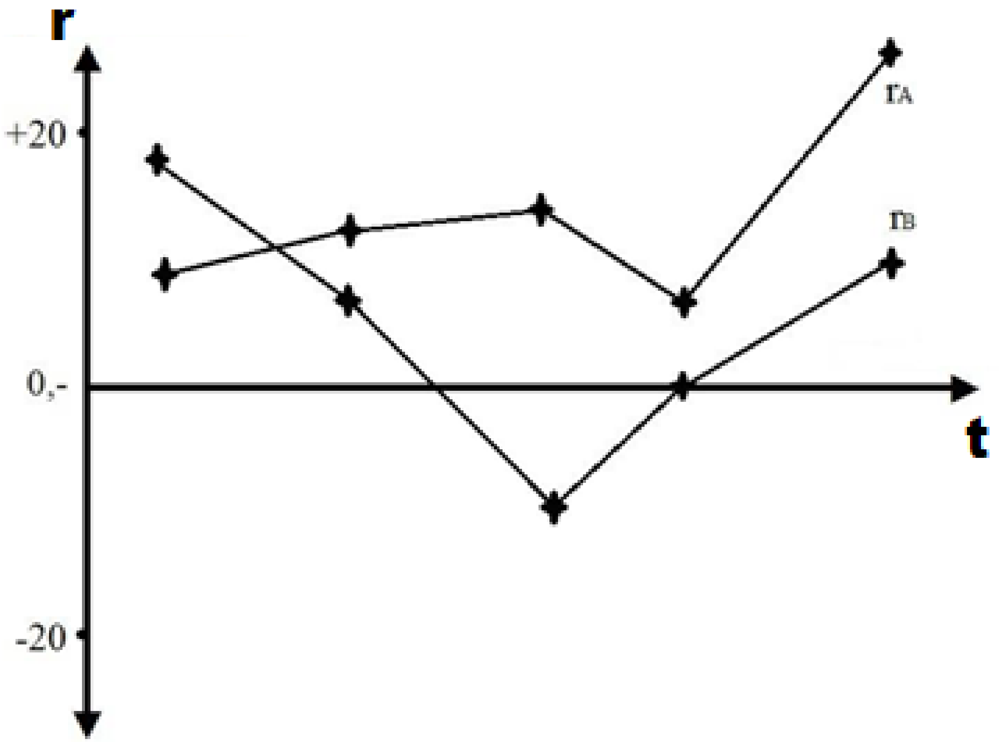
Negatively correlated returns

For them, if one investment instrument has a chance of a high return, the other should be the opposite.

It also applies at the expected low rate of return. To put it simply:

"... if one fails, the other comes ...".

Non-correlated returns



Non-correlated returns

Non-correlated returns

Their rate of return is not related. Their correlation coefficient is close to zero.

Therefore, if an investor wants to diversify portfolio risk, he must invest in negatively correlated assets

Moody's Analytics Asset Level Correlation

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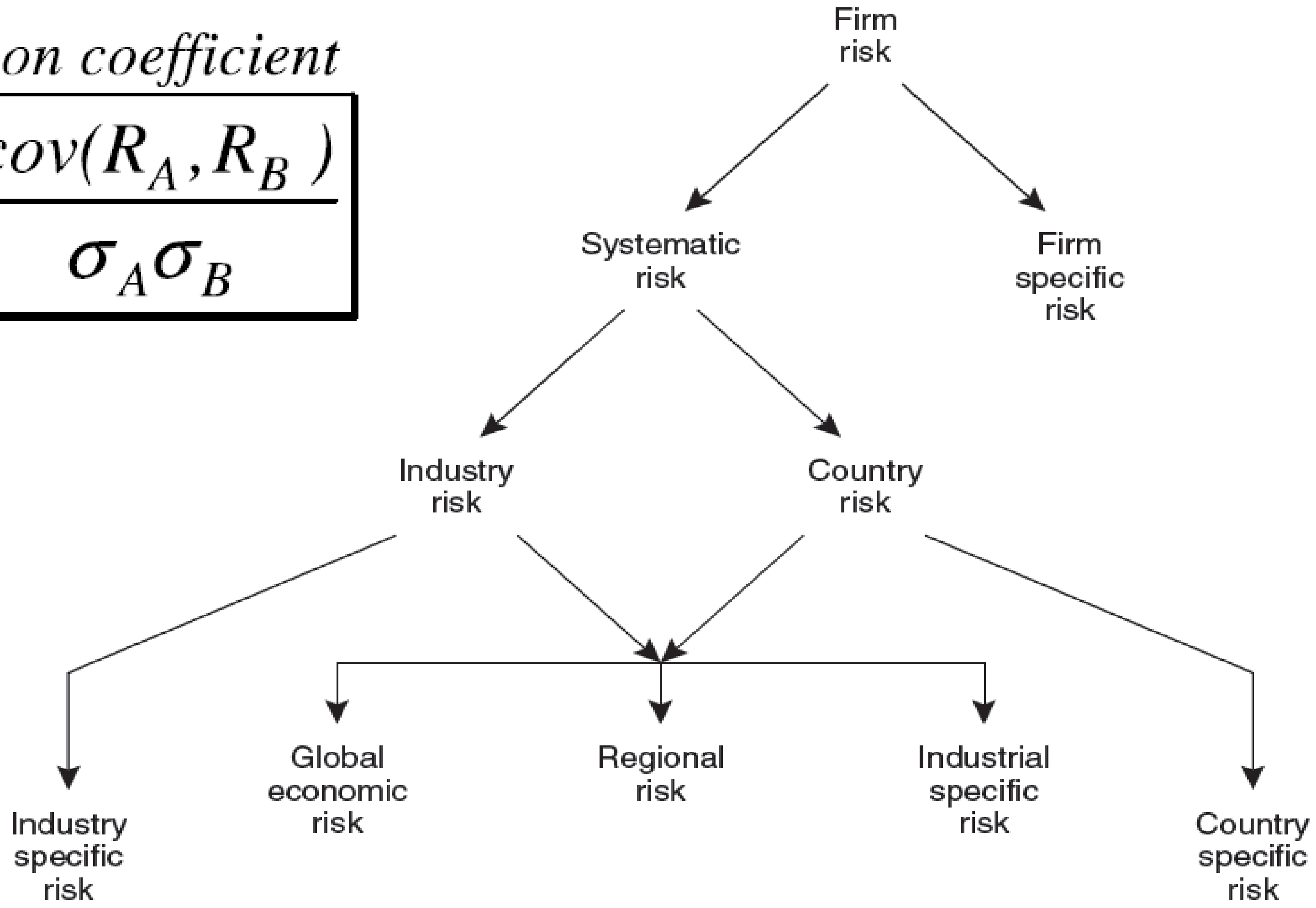
Correlation coefficient

$$\boxed{k_{AB} = \frac{cov(R_A, R_B)}{\sigma_A \sigma_B}}$$



Risk of portfolio

Risk (standard deviation) of portfolio

$$\sigma = \sqrt{\sigma^2} = \sqrt{X^2\sigma_A^2 + (1-X)^2\sigma_B^2 + 2X(1-X)cov(R_A, R_B)}$$

where: $\sigma_p{}^2$ – variance of portfolio of assets A and B
$cov(R_{A,B})$ – covariance of assets A and B
$\sigma_{A,B}$ – standard deviation of asset A and asset B
X – ratio of asset A on portfolio investment

Covariance

$$\boxed{cov(R_A, R_B) = \sum_{i=1}^{n} P_i (R_{iA} - R_A)(R_{iB} - R_B)}$$

Variance of portfolio

$$\sigma_p^2 = X^2\sigma_A^2 + (1-X)^2\sigma_B^2 + 2X(1-X)cov(R_A, R_B)$$

where: $\sigma_p{}^2$ – variance of portfolio of assets A and B
$cov(R_{A,B})$ – covariance of assets A and B
$\sigma_{A,B}$ – standard deviation of asset A and asset B
X – ratio of asset A on portfolio investment

Expected rate of portfolio profit

$$R_p = XR_A + (1-X)R_B$$

where: R_p – expected average rate of portfolio profit (mean)
$R_{A,B}$ – expected average rate of profit (mean) of assets A and B
X – ratio of asset A on portfolio investment

Example: Risk of a portfolio investment

5. Calculate the expected rate of return on the portfolio and the risk associated with the portfolio consisting of the above assets (**we invest CZK 500,000 in Asset A and CZK 250,000 in Asset B**) and determine whether these assets are suitable for building the portfolio. Proof your claim by calculation.

Variant no.	Economic growth	Probability to reach returns (%)	Returns of investments (%)	
			A	B
1	Positive	10	20	12
2	Stable	60	15	10
3	Negative	30	5	15

- Follow the steps as for the previous cases of a simple investment.

You know these steps ☺

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$$\boldsymbol{a})\ \boldsymbol{R_A} = \mathbf{0.1} * 20 + \mathbf{0.6} * 15 + \mathbf{0.3} * 5$$

$$\boldsymbol{R_B} = \mathbf{0.1} * 12 + \mathbf{0.6} * 10 + \mathbf{0.3} * 15$$

$$\boldsymbol{b})\ \sigma_A^2 = \mathbf{0.1}(20 - \boldsymbol{R_A})^2 + \mathbf{0.6}(15 - \boldsymbol{R_A})^2 + \mathbf{0.3}(5 - \boldsymbol{R_A})^2$$

$$\sigma_B^2 = \mathbf{0.1}(12 - \boldsymbol{R_B})^2 + \mathbf{0.6}(10 - \boldsymbol{R_B})^2 + \mathbf{0.3}(15 - \boldsymbol{R_B})^2$$

- But now You have to count the expected return of portfolio with both shares of investment A and investment B.

$$c)\ X = \frac{A}{(A + B)}$$

Expected rate of portfolio profit

$$\boxed{R_p = XR_A + (1 - X)R_B}$$

Further steps are a **covariance** to examine the **variance of portfolio**

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$$d)\ cov_{A,B} = \mathbf{0.1}(20 - \boldsymbol{R_A})(12 - \boldsymbol{R_B}) + \mathbf{0.6}(15 - \boldsymbol{R_A})(10 - \boldsymbol{R_B}) + \mathbf{0.3}(5 - \boldsymbol{R_A})(15 - \boldsymbol{R_B})$$

- A covariance means simply combining a part of calculations (parenthesis) of each variances. Therefore, it is NOT squared! ☺

Covariance

$$\boxed{cov(R_A, R_B) = \sum_{i=1}^{n} P_i (R_{iA} - R_A)(R_{iB} - R_B)}$$

- Finally, You know everything to measure the risk of portfolio ☺ You are GREAT! ☺

Variance of portfolio

$$\boxed{\sigma_p^{\,2} = X^2 \sigma_A^{\,2} + (1 - X)^2 \sigma_B^{\,2} + 2X(1 - X)\operatorname{cov}(R_A, R_B)}$$

Example of compounding the risk of investment assets

a) Calculate expected rate of profit, standard deviation

b) What is the covariance and the correlation coefficient for each pair of shares? Are they suitable for portfolio investment?

c) Calculate expected rate of profit and standard deviation of a portfolio consisting of 60 % A shares and 40 % B shares.

Economic situation	**Probability (%)**	**Rate of profit for shares of different companies (%)**		
		A	**B**	**C**
Growing	20	25	19	20
Average grow	40	21	18	15
Stagnation	30	15	14	10
Recession	10	9	12	10

Example of compounding the risk of investment assets

a) Expected rate of profit:

$$R_A = 0.20 \times 25 + 0.40 \times 21 + 0.30 \times 15 + 0.10 \times 9 = 18.8\ \%$$

$$R_B = 0.20 \times 19 + 0.40 \times 18 + 0.30 \times 14 + 0.10 \times 12 = 16.4\ \%$$

$$R_C = 0.20 \times 20 + 0.40 \times 15 + 0.30 \times 10 + 0.10 \times 10 = 14\ \%$$

Variance and standard deviation:

$$\sigma_A^2 = 0.2(25 - 18.8)^2 + 0.4(21 - 18.8)^2 + 0.3(15 - 18.8)^2 + 0.1(9 - 18.8)^2 = 23.56$$

$$\sigma_A = \sqrt{23.56} = 4.85\ \%$$

$$\sigma_B^2 = 0.2(19 - 16.4)^2 + 0.4(18 - 16.4)^2 + 0.3(14 - 16.4)^2 + 0.1(12 - 16.4)^2 = 6.04$$

$$\sigma_B = \sqrt{6.04} = 2.46\ \%$$

$$\sigma_C^2 = 0.2(20 - 14)^2 + 0.4(15 - 14)^2 + 0.3(10 - 14)^2 + 0.1(10 - 14)^2 = 14$$

$$\sigma_C = \sqrt{14} = 3.74\ \%$$

Example of compounding the risk of investment assets

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b) Covariance for each pair of shares:

$$cov(R_A, R_B) = 0.2(25-18.8)(19-16.4) + 0.4(21-18.8)(18-16.4) + 0.3(15-18.8)(14-16.4) + 0.1(9-18.8)(12-16.4) = 11.68$$

$$cov(R_A, R_C) = 0.2(25-18.8)(20-14) + 0.4(21-18.8)(15-14) + 0.3(15-18.8)(10-14) + 0.1(9-18.8)(10-14) = 16.8$$

$$cov(R_B, R_C) = 0.2(19-16.4)(20-14) + 0.4(18-16.4)(15-14) + 0.3(14-16.4)(10-14) + 0.1(12-16.4)(10-14) = 8.4$$

Correlation coefficient for each pair of shares:

$$k_{A,B} = \frac{11.68}{4.85 \times 2.46} = 0.98 \rightarrow not\ suitable\ for\ portfolio\ investment$$

$$k_{A,C} = \frac{16.8}{4.85 \times 3.74} = 0.93 \rightarrow not\ suitable\ for\ portfolio\ investment$$

$$k_{B,C} = \frac{8.4}{2.46 \times 3.74} = 0.91 \rightarrow not\ suitable\ for\ portfolio\ investment$$

Example of compounding the risk of investment assets

c) Expected rate of profit and risk of portfolio (60 % A and 40 % B):

$$R_p = 0.6 \times 18.8 + (1 - 0.6)16.4 = 17.84\ \%$$

$$\sigma_p^2 = 0.6^2 \times 23.56 + (1 - 0.6)^2 \times 6.04 + 2 \times 0.6(1 - 0.6)11.68 = 15.05$$

$$\sigma_p = \sqrt{15.05} = 3.88\ \%$$

Average expected rate of portfolio, consisting of 60 % shares A and 40 % shares B, is 17.87 % with risk 3.88 %.

Literature:

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Thank you for your attention!

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