

Derivace a integrace

Funkce $f : y = f(x)$

$f'(x)$

$\int f(x)dx = F(x) + c$

Základní jsou vzorce s šedým pozadím

$$(af(x))' = af'(x)$$

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\left[\frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

derivace složené funkce

$$\int af(x)dx = a \int f(x)dx$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

Na integraci jiných operací musíme použít metody: per partes nebo substituční

Pravidla

$$y = k \text{ (konstanta)}$$

$$(k)' = 0$$

$$\int k dx = kx + c$$

$$y = x^n, n \in \mathbb{N} \dots\dots$$

$$(x^n)' = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$y = \frac{1}{x}$$

$$\left(\frac{1}{x} \right)' = (x^{-1})' = -x^{-2} = -\frac{1}{x^2}$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$y = e^x$$

$$(e^x)' = e^x$$

$$\int e^x dx = e^x + c$$

$$y = a^x (a > 0, a \neq 1)$$

$$(a^x)' = a^x \ln a$$

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$y = \sin x$$

$$(\sin x)' = \cos x$$

$$\int \sin x dx = -\cos x + c$$

$$y = \cos x$$

$$(\cos x)' = -\sin x$$

$$\int \cos x dx = \sin x + c$$

$$y = \operatorname{tg} x$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$$\int \operatorname{tg} x dx = -\ln|\cos x| + c$$

$$y = \operatorname{cot} gx$$

$$(\operatorname{cot} gx)' = \frac{-1}{\sin^2 x}$$

$$\int \operatorname{cot} gx dx = \ln|\sin x| + c$$

$$y = \frac{1}{\cos^2 x}$$

$$(\cos^{-2} x)' = 2 \cos^{-3} x \sin x$$

$$\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + c$$

$$y = \frac{1}{\sin^2 x}$$

$$(\sin^{-2} x)' = 2 \sin^{-3} x \cos x$$

$$\int \frac{1}{\sin^2 x} dx = -\operatorname{cot} gx + c$$

$$y = \operatorname{arctg} x$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

integrujeme metodou per partes

$$y = \operatorname{arcsin} x$$

$$(\operatorname{arcsin} x)' = \frac{1}{\sqrt{1-x^2}}$$

integrujeme metodou per partes

$$y = \frac{1}{\sqrt{1-x^2}}$$

$$\left(\frac{1}{\sqrt{1-x^2}} \right)' = \frac{x}{\sqrt{(1-x^2)^3}}$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \operatorname{arcsin} x + c$$

$$y = \frac{1}{1+x^2}$$

$$\left(\frac{1}{1+x^2} \right)' = \frac{-2x}{(1+x^2)^2}$$

$$\int \frac{1}{1+x^2} dx = \operatorname{arctg} x + c$$

Goniometrické vzorce

$$\operatorname{tg} x \operatorname{cotg} x = 1,$$

$$\sin^2 x = \frac{1 - \cos 2x}{2},$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}.$$

$$\sin^2 x + \cos^2 x = 1,$$

$$\sin 2x = 2 \sin x \cos x,$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x,$$

Mocniny:

- $a^n \cdot a^m = a^{n+m}$ např.: $(a^2 \cdot a^3 = a^5)$
- $\frac{a^n}{a^m} = a^{n-m}$ např.: $a^5/a^3 = a^2$
- $\frac{a^n}{a^n} = a^0 \Rightarrow a^0 = 1$
- $\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n}$ např.: $1/a^3 = a^{-3}$
- $(a^n)^m = a^{n \cdot m}$ např.: $(a^2)^3 = a^6$
- $(a \cdot b)^n = a^n \cdot b^n$ např.: $(a \cdot b)^2 = a^2 \cdot b^2$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ např.: $(a/b)^2 = a^2/b^2$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$ např.: $a^{2/3} = \sqrt[3]{a^2}$

Logaritmy:

- $\ln e = 1$
- $\ln 1 = 0$
- $\ln x + \ln y = \ln(xy)$
- $\ln x - \ln y = \ln \frac{x}{y}$
- $\ln x^y = y \ln x$ např.: $\ln \sqrt{x} = \frac{1}{2} \ln x$
- $e^{\ln A} = A$ např.: $e^{\ln x} = x$

Neurčité výrazy

$$\frac{\infty}{\infty}; \frac{0}{0}; \infty - \infty; 0 \cdot \infty; 0^\infty; 1^\infty; \infty^0;$$

Počítání s nekonečnem

$$\infty + \infty = \infty, \infty \cdot \infty = \infty, \infty^\infty = \infty$$

$$\frac{a}{\infty} = 0, \frac{a}{-\infty} = 0, \frac{a}{0} = \pm\infty, \frac{0}{a} = 0$$

Diferenciál: $dy = y' dx$,

$$\text{Totální diferenciál: } dz = z'_x dx + z'_y dy,$$

$$d^2 z = z''_{xx} dx^2 + 2z''_{xy} dx dy + z''_{yy} dy^2$$

Taylorův rozvoj (polynom) funkce $f(x)$ v okolí bodu a .

$$T_n(f, a, x) = f(a) + \frac{f'(a)}{1!} (x-a) + \frac{f''(a)}{2!} (x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!} (x-a)^n$$

$$\text{Derivace funkce } F(x,y)=0: y' = -\frac{\partial_x F}{\partial_y F}$$

Vzorce zkráceného násobení:

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Základní limity:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = e^k, \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{k} = 1, \lim_{n \rightarrow \infty} \sqrt[n]{n!} = \infty$$

Odmocniny:

$$1. \sqrt[n]{a \cdot b} = (a \cdot b)^{\frac{1}{n}} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$2. \sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$3. \sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad 4. \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Definiční obory elementárních funkcí:

$$y = \ln x, x > 0,$$

$$y = \sqrt{x}, x \geq 0,$$

$$y = \arcsin x, -1 \leq x \leq 1,$$

$$y = \arccos x, -1 \leq x \leq 1,$$

$$y = \frac{f(x)}{g(x)}, g(x) \neq 0.$$