SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

Mathematics in economics

Lecture 11

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Differential equations - continued

Now assume that q(x) is not zero:

$$y' + \quad =$$

In such case we use the method called variation of parameters. We assume the solution of the form:

$$y = \quad \int$$

But C is now a function:

$$y = \quad (\ ) \quad \int$$

Differential equations - continued

Substituting the last formula into $y' + \quad = \quad$ yields the solution.

Example: $y' + \quad =$

Solution: we search for a solution of the form $y = \quad {}^{x^2}$.
Substituting into the equation:

$$c'(x)e^{-x^2} + \quad - + \quad \cdot =$$

$$c'(x)e^{-x^2} =$$

Differential equations - continued

Rearranging of terms yields:

$$c'(x) = \quad x^2$$

Now we integrate: $c(x) = \int \quad x^2 \quad x^2 \quad +$

Solution of the given equation is:

$$y = + \quad x^2$$

Differential equations – Problem 1

Solve: $y' - \frac{1}{x+2}y = \quad .$

Solution:

First, we solve corresponding homogenous equation by the separation of variables method:

$$\frac{dy}{dx} = \frac{1}{x+2}y$$

$$\frac{dy}{y} = \frac{dx}{x+2}$$

$$\ln|y| = \ln|x+2| + C$$

And finally:

$$y = C(x+2)$$

Differential equations – Problem 1 – cont.

In the second step, we apply the variation of a constant method:

$$y = C(x).x +$$

Substitution:

$$C'(x)(x+2) + C(x).1 - \frac{1}{x+2}C(x).(x+2) =$$

$$C'(x).(x+2) =$$

$$C'(x) = \frac{x}{x+2}$$

$$C(x) = \int \frac{x}{x+2}dx = \int \frac{x+2}{x+2}dx + \int \frac{(-2)}{x+2}dx = x - 2\ln|x+2| + C$$

$$y = (x+2)[x - 2\ln|x+2| +$$

Linear differential equations of the second order with constant coefficients

The last type of differential equation we will address.

It is of the form: $a\dot{y} + by + cy =$

A solution is assumed to be in the form $y =$
where lambda is a root of the so called characteristic equation:

$$a\lambda \quad b\lambda \quad =$$

Linear differential equations of the second order with constant coefficients

In the aforementioned three cases, we yield following solutions:

Case 1: $y = \quad + \quad$

Case 2: $y = \quad + \quad$

Case 3: $y = \quad + \quad$

Linear differential equations of the second order with constant coefficients

The characteristic equation is a quadratic equation, which means we have three cases:

- Two real roots.
- One real root of the order two.
- Two imaginary roots.

Linear differential equations of the second order with constant coefficients - Problem 1

Solve: $y'' - \quad + \quad =$

Solution: we start with the characteristic equation:

$$\lambda \quad \lambda \quad -$$

This equation has two real roots: $\lambda_1 = 2$ and $\lambda_2 = 3$.

Therefore, th e solution is:

$$y = \quad e^{2x} + \quad e^{3x}$$

Linear differential equations of the second order with constant coefficients - Problem 2

Solve: $y' - y - y =$

Solution: we start with the characteristic equation:

$$\lambda \quad \lambda \quad -$$

This equation has two real roots: $\lambda_1 = 3$ and $\lambda_2 = -1$.

Therefore, th e solution is:

$$y = \quad e^{-} + \quad e^{3x}$$

Linear differential equations of the second order with constant coefficients - Problem 3

Solve: $y'' - \quad + \quad =$

Solution: we start with the characteristic equation:

$$\lambda - \quad + -$$

This equation has two real roots: $\lambda_1 = 3$ and $\lambda_2 = 3$.

Therefore, th e solution is:

$$y = \quad +$$

Linear differential equations of the second order with constant coefficients - Problem 4

Solve: $y'' + \quad + \quad =$

Solution: we start with the characteristic equation:

$$\lambda \quad + \quad + \quad =$$

This equation has two real roots: λ_1 = -2+i and λ_2 = -2-i.

Therefore, th e solution is:

$$y = \qquad + \qquad$$

Linear differential equations of the second order with constant coefficients - Problem 5

Solve: $y'' - y =$

Solution: we start with the characteristic equation:

$$\lambda \quad -$$

This equation has two real roots: $\lambda_1 = 1$ and $\lambda_2 = -1$.

Therefore, th e solution is: $y = \; e^{-} + \; e^{x}$

Linear differential equations of the second order with constant coefficients - Problem 6

Solve: $y' + y =$

Solution: we start with the characteristic equation:

$$\lambda \quad -$$

This equation has two real roots: $\lambda_1 = i$ and $\lambda_2 = -i$.

Therefore, the solution is:

$$y = \sin x + \cos x$$

Linear differential equations of the second order with constant coefficients

Now we will focus on eqautions with non-zero right hand side:

$$ay + \quad + \quad = \quad x)$$

$$a, b, c \in R \quad a \neq$$

This type of equation is called non-homogenous.

Solution of this equation has the following form:

$$C_1 y_1 + \quad y_2 + \quad x)$$

Linear differential equations of the second order with constant coefficients – cont.

The solution $C_1 y_1 + y_2$ correspond to a homogenous case, while $P(x)$ is the so called particular integral, which solves a nonhomogenous part of an equation.

A particular integral for the most common functions (polynomials, exponentials, logarithms, etc.) can be easily “guessed“.

We will illustrate the procedure by several examples.

Linear differential equations of the second order with constant coefficients – Problem 9

Solve: $y' - \; - \; =$

Solution: we begin with the homogenous case and its characteristic polynom:

$$\lambda \quad \lambda \quad -$$

The roots are $\lambda_1 = 2$ a $\lambda_2 = -1$, hence the solution is:

$$y = {}_1 e^{2x} + {}_2 e^{-}$$

Now we seek a particular integral in the form:

$$H(x) = \quad +$$

Linear differential equations of the second order with constant coefficients – Problem 9 – cont.

Solve: $y' - \; - \; =$

Solution: we substitute y = P(x) into the given equation:

$$- \; - \; x + \; =$$

Which yields: a = -2, b = 2.

Therefore, the general solution to the equation is:

$$y = Ce^{2x} + Ce^{-x} - x +$$

Linear differential equations of the second order with constant coefficients – Problem 10

Solve: $y'' + \quad = \quad -$

Solution: we begin with the homogenous case and its characteristic polynom:

$$\lambda \quad \lambda$$

The roots are $\lambda_1 = 0$ a $\lambda_2 = -4$, hence the solution is:

$$y = \quad + \quad e^{-}$$

Now we seek a particular integral in the form:

$$P(x) = \quad + \quad + \quad = \quad + \quad +$$

Linear differential equations of the second order with constant coefficients – Problem 10 – cont.

Solve: $y'' - \quad - \quad =$

Solution: we substitute y = P(x) into the given equation:

$$6ax + \quad + \quad + \quad + \quad = \quad -$$

Which yields: $a = \quad = - \quad = -$

Therefore, the general solution to the equation is:

$$C_1 + \quad e^{-} \quad + \quad x^3 - \quad x^2 - \quad 7x$$

Problems to solve - 1

Solve:

$$y' - x + y =$$

$$y' - \frac{y}{2x} =$$

$$y' + \frac{y}{x} =$$

$$y' + \quad =$$

Problems to solve - 2

Solve:

$$y' + y - y =$$

$$2y' - y' =$$

$$y' + y + y =$$

$$y' - y + y =$$

$$y' + y + y =$$

Problems to solve - 3

Solve:

$$y + 3 + 2 =$$

$$y'' + + = + +$$

$$y'' + + =$$

$$y'' + - =$$

Final remarks

- See the exam dates in STAG. Everybody has 2 attempts.
- Also, see the older versions of exam tests on my public or Moodle.
- If you need consultations, write me (or Dr. Stoklasova) an e-mail.
- Good luck!

Thank you for your attention!