SILESIAN UNIVERSITY

SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

Mathematics in economics

Lecture 10

Definite integral

Newton s definite integral:

$$\int^{b} \qquad - $$

In the definition above, F is a primitive function to f,
and *a* and *b* are the limits of the integral.

The result of definite integral is not a function, but a number!

Definite integral – elementary properties

Generally, when computing definite integral, we use the same table of elementary integrals as for an indefinite integral.

The elementary properties of the definite integral:

$$\int^{a}$$

$$\int^{b} \qquad \int^{a}$$

$$\int^{b} \qquad \int^{b}$$

$$\int^{b} \qquad \int^{b} \qquad \int^{b}$$

$$\int^{c} \qquad \int^{b} \qquad \int^{c}$$

Definite integral – a use

The definite integral can be used to calculation of:

- The square under or above given function,
- The area of a 2D object,
- The length of a curve,
- The volume of a 3D object,
- The area of a 3D object.

Definite integral – An area under/above a function

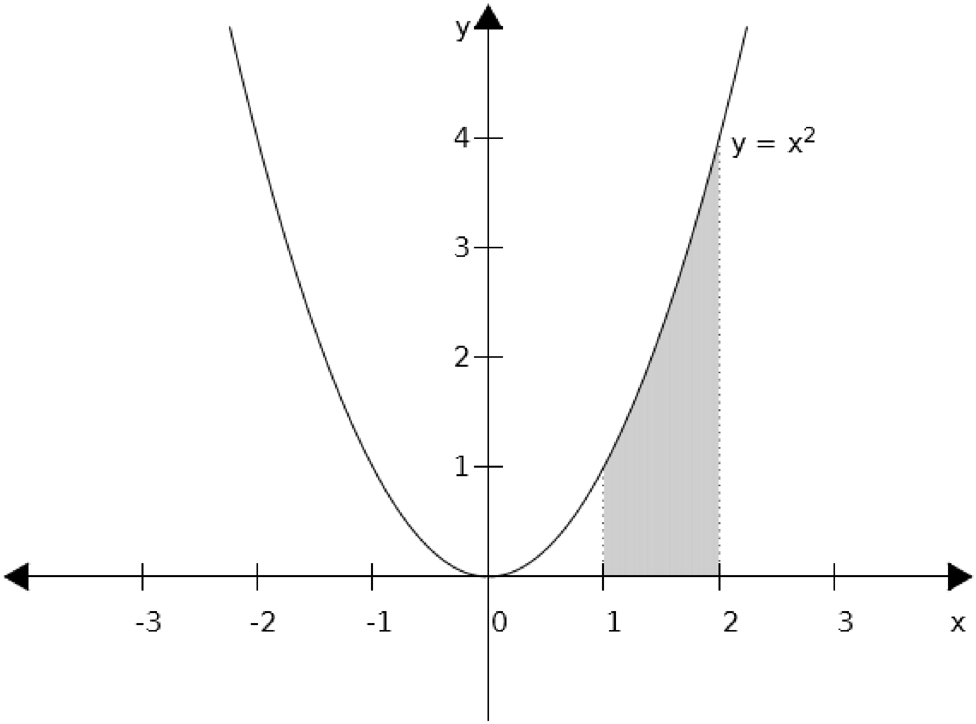
Find: $\int^{2}$

Solution: $\int^{2} \quad \left[\quad \right]^{2} \quad 3 \quad 7 \quad =$

What does the number 7/3 mean?

It is the area below the function f(x) on the interval (1,2),
See the next slide for a picture.

Definite integral – An area under/above a function



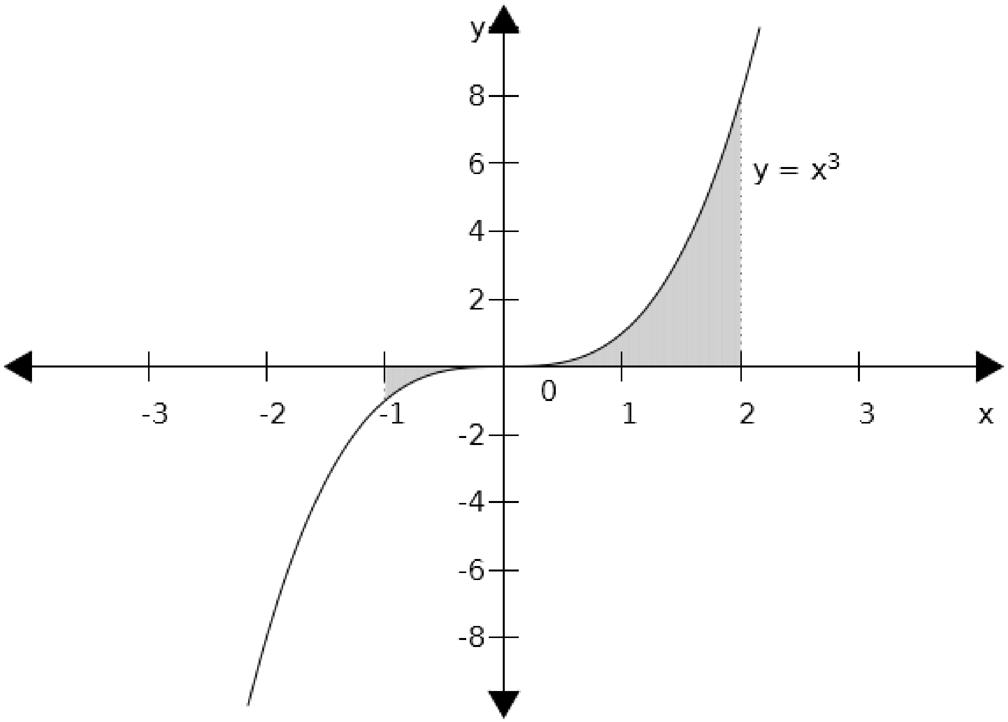
Definite integral – An area under/above a function

Find an area bounded by functions: $y =$, axis x, x = -1 and x = 2.

Solution: We must divide the interval of integration (-1,2) into two intervals: (-1,0) and (0,2) (WHY?):

$$S = \int^{0} \qquad \int^{2} \qquad \qquad + \quad - \quad = \quad 1 \quad + \quad = \quad 17$$

Definite integral – An area under/above a function



Definite integral – An area under/above a function
Problem 2

Find: $\int^{3}$

Solution: $\int^{3} \; = $

This result means that the area under the function on the interval (0,3) is 9.

Important note: if a function is positive on the interval of integration, then the result will be a positive number. However, for a negative function the result will be negative!

Definite integral – An area under/above a function
Problems 3 and 4

Find: $\int$

Solution: $\int \quad = - \quad - - \quad = + =$

Find: $\int$

Solution: $\int \quad = \left\lfloor \quad \right\rfloor \quad - \quad = - =$

Problems to solve - 1

Find: $\int^{4}$

$\int^{4}$

$\int^{3}$

e

$\int$

$\int$

An area between two curves

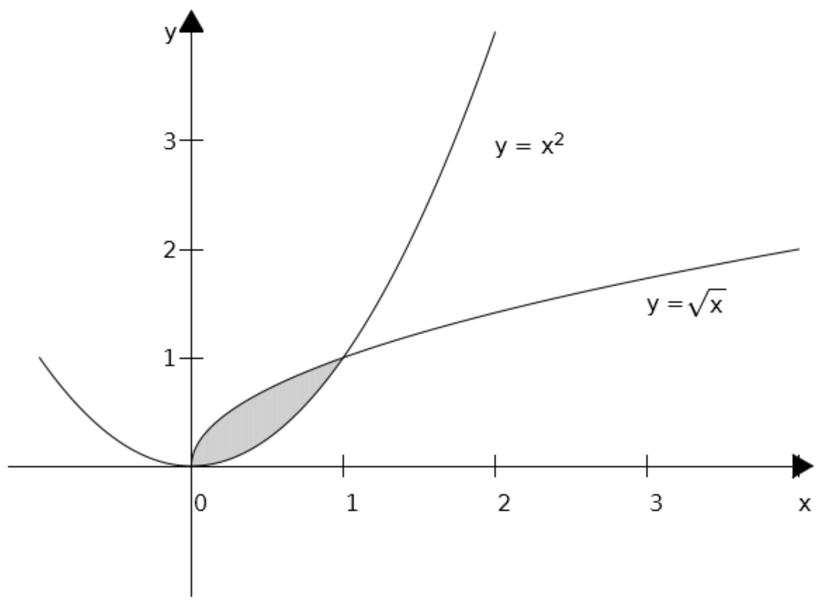
Let f(x) and h(x) be two curves, *S* an area between them
And *a* and *b* their intersections.
Then *S* is given as follows:

$$S = \int^{b}$$

An area between two curves – Problem 1

Find an are between two curves: $y =$ and $y =$.

A picture:



An area between two curves – Problem 1 cont.

Find an are between two curves: $y =$ and $y =$.

Solution:
First, we find intersections: $x^2 =$, hence x = 0 and x = 1.

Now, we can use the integral formula for the area:

$$S = \int^{1} \quad = \left[\quad \right]^{1} \left(\quad \right) \left(\quad \right)$$

An area between two curves – Problem 2

Find an are between two curves: $y =$ and $y =$.

Solution:
First, we find intersections: $x^2 =$, hence x = 0 and x = 2.

Now, we can use the integral formula for the area:

$$S = \int^{2}$$

An area between two curves – Problem 2 – cont.

Find an are between two curves: $y =$ and $y =$.

A graph: