Mathematics in Economics – lecture 3

1) Composite derivative (Chain rule)

In simple words, we say that the derivative of a composite function is **the product of the derivative of the outside function with respect to the inside function and the derivative of the inside function with respect to the variable**.

$$y = \ln(4x+1)$$

$$y = (x^3 + 4x^2)^5$$

$$y = \sqrt{x^2 + 4x}$$

2) The second derivative

The derivative of a function y = f(x) of a variable x is **a measure of the rate at which the value y of the function changes with respect to the change of the variable x**. It is called the derivative of f with respect to x.

The second derivative is **the rate of change of the rate of change of a point at a graph** (the "slope of the slope" if you will). This can be used to find the acceleration of an object (velocity is given by first derivative).

If a function f′(x) can be differentiated, we obtain the second derivative of f(x), denoted as f′′(x), and so on.

a) $y = 3x^4 + 2x^2 - x + 1$ Find $y''(2) =$

b) $y = 4x^3 + 5x + 1$ Find $y'''(1) =$

c) $y = -5x^4 + 3x^3 + 1$ Find $y'''(0) =$

3) Taylor and Maclaurin series

The **Taylor series** of a function is an infinite sum of terms that are expressed in terms of the function's derivatives at a single point. For most common functions, the function and the sum of its Taylor series are equal near this point. Taylor series are named after Brook Taylor, who introduced them in 1715.

If 0 is the point where the derivatives are considered, a Taylor series is also called a **Maclaurin series**, after Colin Maclaurin, who made extensive use of this special case of Taylor series in the 18th century.

Let a function $y = f(x)$ be differentiable of the order n at a point a, then it can be approximated by the Taylor series of the form:

$$T_n(f,a,x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ \dots \ + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_{n+1}(x)$$

- If $a = 0$, we obtain a special case of the Taylor series, called Maclaurin series:

$$T_n(f,0,x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ \dots \ + \frac{f^{(n)}(0)}{n!}x^n + R_{n+1}(x)$$

Maclaurin series of selected functions

Function	Maclaurin series
sinx	$x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots$
cosx	$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
exp(x)	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

a) Find the Taylor series of the function $f(x) = 3x^3 + 2x^2 - 10x + 2$ at the point $a = 2$.

b) Find the Maclaurin series of the function $f(x) = 2x^4 + 3x^2 - 6x + 3$.