MATEMATIKA v EKONOMII – seminář č. 4 –Funkce dvou proměnných

1. Určete definiční obor funkcí dvou proměnných:

a) $f(x,y)=\sqrt{x^2+y^2-4}$ b) $f(x,y)=\sqrt{-x^2+2x-y^2-8y-8}$

c) $f(x,y)=\ln(x^2-4y)$ d) $f(x,y)=\ln(2x+y-1)$

e) $f(x,y)=x+\arccos y$ f) $f(x,y)=\dfrac{5}{x-y}+\dfrac{x}{y}$

g) $f(x,y)=\sqrt{x+y}+\sqrt{y-3}$ h) $f(x,y)=\dfrac{\ln(xy^2)}{x-y}$

Viz zvlášť naskenovaný soubor

2. Určete parciální derivace funkcí:

a) $f(x,y)=x^2+2y^2$ b) $f(x,y)=yx^2+\cos y$ c) $f(x,y)=\sqrt{x^2+y^2+5}$

d) $f(x,y)=\ln(xy+y^4)$ e) $f(x,y)=x\ln(y+x)$ f) $f(x,y)=\sin(xy)$

Výsledky:

a) $\dfrac{\partial f}{\partial x}=2x$, $\dfrac{\partial f}{\partial y}=4y$ b) $\dfrac{\partial f}{\partial x}=2xy$, $\dfrac{\partial f}{\partial y}=x^2-\sin y$, c) $\dfrac{\partial f}{\partial x}=\dfrac{x}{\sqrt{x^2+y^2+5}}$, $\dfrac{\partial f}{\partial y}=\dfrac{y}{\sqrt{x^2+y^2+5}}$, d) $\dfrac{\partial f}{\partial x}=\dfrac{y}{xy+y^4}=\dfrac{1}{x+y^3}$, $\dfrac{\partial f}{\partial y}=\dfrac{x+4y^3}{xy+y^4}$, e) $\dfrac{\partial f}{\partial x}=\ln(x+y)+\dfrac{x}{x+y}$, $\dfrac{\partial f}{\partial y}=\dfrac{x}{x+y}$, f) $\dfrac{\partial f}{\partial x}=y\cos xy$, $\dfrac{\partial f}{\partial y}=x\cos xy$

3. Vypočtěte parciální derivace prvních a druhých řádů funkce

a) $f(x,y)=x^2+y^2+1$ b) $f(x,y)=x^3+2x^2y^2+x$

c) $f(x,y)=\ln xy$ d) $f(x,y)=arctg\,\dfrac{x}{y}$

Výsledky:

a) $\dfrac{\partial f}{\partial x}=2x$, $\dfrac{\partial f}{\partial y}=2y$, $\dfrac{\partial^2 f}{\partial x^2}=2$, $\dfrac{\partial^2 f}{\partial y^2}=2$, $\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{\partial^2 f}{\partial y\partial x}=0$

b) $\dfrac{\partial f}{\partial x}=3x^2+4xy^2+1$, $\dfrac{\partial f}{\partial y}=4x^2y$, $\dfrac{\partial^2 f}{\partial x^2}=6x+4y^2$, $\dfrac{\partial^2 f}{\partial y^2}=8x$, $\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{\partial^2 f}{\partial y\partial x}=8xy$

c) $\dfrac{\partial f}{\partial x}=\dfrac{1}{x}$, $\dfrac{\partial f}{\partial y}=\dfrac{1}{y}$, $\dfrac{\partial^2 f}{\partial x^2}=-\dfrac{1}{x^2}$, $\dfrac{\partial^2 f}{\partial y^2}=-\dfrac{1}{y^2}$, $\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{\partial^2 f}{\partial y\partial x}=0$

d) $\dfrac{\partial f}{\partial x}=\dfrac{y}{x^2+y^2}$, $\dfrac{\partial f}{\partial y}=-\dfrac{x}{x^2+y^2}$, $\dfrac{\partial^2 f}{\partial x^2}=-\dfrac{2xy}{(x^2+y^2)^2}$, $\dfrac{\partial^2 f}{\partial y^2}=\dfrac{2xy}{(x^2+y^2)^2}$,

$\dfrac{\partial^2 f}{\partial x\partial y}=\dfrac{\partial^2 f}{\partial y\partial x}=\dfrac{x^2-y^2}{(x^2+y^2)^2}$

4. Vypočtěte parciální derivace prvního a druhého řádu v bodě C:

a) $f(x,y)=x^2+5y^2+x$, C [1,2] b) $f(x,y)=x^3y^2+y^2$, C [-2,3]

Výsledky: a) $\frac{\partial f}{\partial x}(1,2)=3$, $\frac{\partial f}{\partial y}(1,2)=20$, $\frac{\partial^2 f}{\partial x^2}(1,2)=2$, $\frac{\partial^2 f}{\partial y^2}(1,2)=10$, $\frac{\partial^2 f}{\partial x \partial y}(1,2)=\frac{\partial^2 f}{\partial y \partial x}(1,2)=0$

b) $\frac{\partial f}{\partial x}(-2,3)=108$, $\frac{\partial f}{\partial y}(-2,3)=-42$, $\frac{\partial^2 f}{\partial x^2}(-2,3)=-108$, $\frac{\partial^2 f}{\partial y^2}(-2,3)=-14$, $\frac{\partial^2 f}{\partial x \partial y}(-2,3)=\frac{\partial^2 f}{\partial y \partial x}(-2,3)=72$