Normal distribution

The normal distribution, also known as the Gaussian distribution, is one of the most fundamental and widely studied probability distributions in statistics. It is characterized by its bell-shaped curve when plotted, with the mean (average) at the center and symmetric tails extending indefinitely in both directions. The shape of the curve is determined by two parameters: the mean (μ) and the standard deviation (σ).

Here are some key properties and characteristics of the normal distribution:

1. **Symmetry**: The normal distribution is symmetric around its mean. This means that the probability of observing a value to the left of the mean is the same as the probability of observing a value to the right of the mean.

2. **Central Limit Theorem**: One of the most important properties of the normal distribution is its connection to the Central Limit Theorem (CLT). According to the CLT, the sum (or average) of a large number of independent, identically distributed random variables will be approximately normally distributed, regardless of the underlying distribution of the individual variables. This property makes the normal distribution applicable to a wide range of real-world phenomena.

3. **68-95-99.7 Rule**: In a normal distribution, approximately 68% of the data falls within one standard deviation of the mean, approximately 95% falls within two standard deviations, and approximately 99.7% falls within three standard deviations.

4. **Probability Density Function (PDF)**: The probability density function of the normal distribution is given by the formula:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

   where x represents a random variable, μ is the mean, σ is the standard deviation, and π is the mathematical constant pi.

4. **Standard Normal Distribution**: A special case of the normal distribution with a mean of 0 and a standard deviation of 1 is known as the standard normal distribution. It is often denoted by Z and is widely used in statistical tables and calculations.

5. **Applications**: The normal distribution is widely used in various fields, including finance, physics, engineering, social sciences, and many others. It serves as a fundamental tool for statistical inference, hypothesis testing, and modelling real-world phenomena.

Overall, the normal distribution plays a central role in statistics and provides a powerful framework for understanding and analysing random phenomena in diverse fields of study.

Exponential distribution

The exponential distribution is a probability distribution that describes the time between events in a Poisson process, where events occur continuously and independently at a constant average rate. It is characterized by its probability density function (PDF), which is skewed to the right and has a heavy tail on the left side.

Here are some key properties and characteristics of the exponential distribution:

1. **Probability Density Function (PDF)**: The probability density function of the exponential distribution is given by the formula:

$$f(x) = \lambda e^{-\lambda x}$$

where x represents a random variable, and λ (lambda) is the rate parameter, which is the average number of events per unit of time.

2. **Memoryless Property**: One of the most important properties of the exponential distribution is its memorylessness. This means that the probability of an event occurring in the next instant of time does not depend on how much time has already elapsed. Mathematically, it can be expressed as:

$$P (X > s+t \mid X > s) = P (X > t)$$

where X is a random variable following an exponential distribution, and s and t are non-negative real numbers representing time intervals.

3. **Expected Value and Variance**: The expected value (mean) of an exponential distribution is 1/λ, and its variance is $\frac{1}{\lambda^2}$.

4. **Applications**: The exponential distribution is commonly used in various fields, including reliability engineering, queueing theory, telecommunications, and survival analysis. It is particularly useful for modelling the time until the occurrence of events, such as the lifespan of electronic components, waiting times in service systems, and the time until the next radioactive decay.

5. **Survival Function**: The survival function of the exponential distribution represents the probability that an event will occur after a certain amount of time. It is given by:

$$S(x) = e^{-\lambda x}$$

6. **Exponential Family**: The exponential distribution is a member of the exponential family of distributions, which includes several other distributions with similar properties.

Overall, the exponential distribution provides a useful framework for modelling random processes characterized by constant rates of occurrence and is widely applied in various practical and theoretical contexts.