SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

Mathematics in economics

Lecture 8

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Mathematics in Economics/PMAT

Application of calculus in economics
Consumer surplus and producer surplus

Consumer surplus (CS) and producer surplus (PS) under perfect competition is given as follows:

$$CS = \int^{Q_E}$$

$$PS = \quad - \int^{Q_E}$$

Consumer surplus and producer surplus
Problem 1

Find CS and PS if $S(Q) = \; + \;$ and $D(Q) = \frac{\;}{\; - \; + \;}$.

Solution:

In the equilibrium demand equals supply:

$$4 + \; = \frac{\;}{\; - \; + \;}$$

From this quadratic equation we obtain two roots:
Q = 5 and Q = -10.
However, the negative root is implausible, so Q_E=5.

Consumer surplus and producer surplus
Problem 1 – cont.

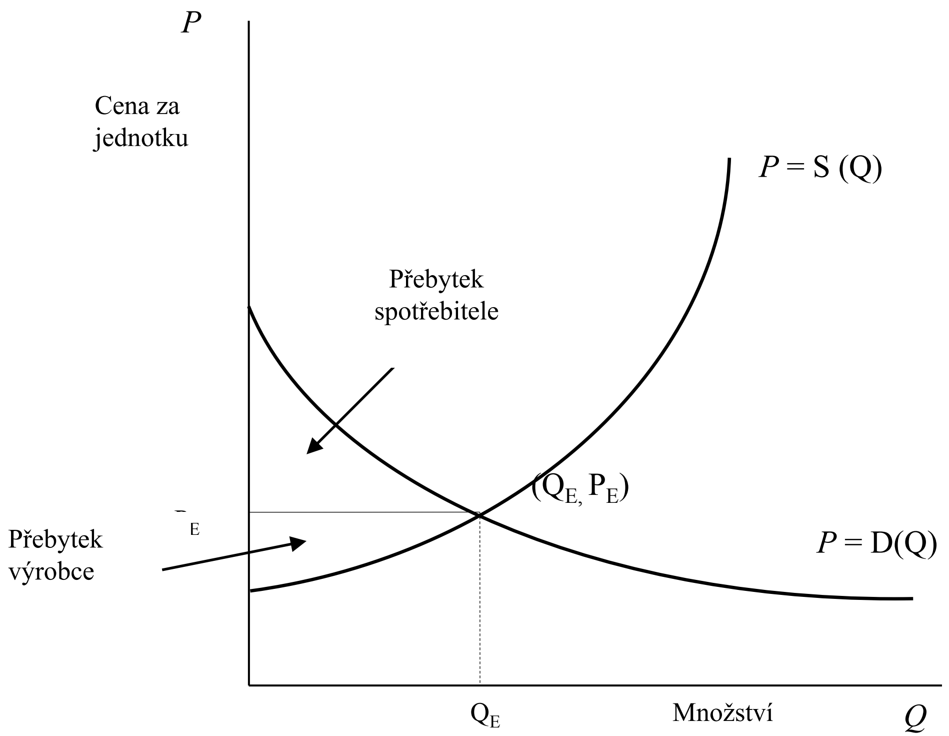
Now, we can found CS and PS with the use of aforementioned integral formulas:

$$CS = \int^{5} \quad = \Big[\quad \Big] \quad - \quad =$$

$$PS = \quad - \int^{5} \quad = \Big[\quad \Big]^{5} \quad =$$

Note, that results must be positive.

Consumer surplus and producer surplus



A triangular area (upper arrow) is CS, lower arrow depicts PS.

A total revenue as an integral of an intensity

In some cases, a total revenue (TR) is given as a sum of an intensity (or a flow) f (t) during a given period of time:

$$TR = \int^{t_2}$$

Example: Find the total revenue for the intensity $f(t) =$
From 1 to 15 years.
Solution:

$$TR = \int^{15} \qquad \int^{15} \qquad \Big]^{15} \qquad - \qquad =$$

= 324996 CZK.

Infinite number series

An infinite number series is given and denoted as follows:

$$a_1 + \quad + \quad + \; + \quad = \sum^{n}$$

It is an infinite sum of numbers a_i.

• If the sum above (a series) has a finite sum, it is called *convergent*.

• Otherwise (when the sum is infinite or does not exist) a series is called *divergent*.

Infinite number series

The most important problems are:

- Is a series convergent or divergent?

- If a series is convergent, what is its sum?

A historical note: recall the paradox of Achilleus and a turtle. In ancient Greece, it was assumed that a sum of an infinite series is always infinite.

Infinite number series
Introduction example

Consider a division of a pizza so that in the first step ½ of pizza is eaten. In the second step ½ of what remained Is eaten, etc.

This situation can be rewwritten as the infinite series:

$$\frac{1}{2} + \quad + \quad + \quad +$$

Unsurprisingly, although the series is infinite, its sum is 1.

Infinite number series
Another examples

Find the following sum (Grandi series):

$$\sum \quad + - + -$$

This sum does not exist.

Find the following sum (harmonic series):

$$\sum \quad + + +$$

The sum is infinite.

Infinite number series
A necessary condition for convergence

A necessary condition for convergence can be stated as follows:

$$\lim_{n \to \infty} a_n =$$

However, this condition is not sufficient, as harmonic series fullfils the condition above, but its sum is infinite.

Convergence or divergence is decided upon a suitable convergence criterion.

Infinite number series
Criterions (tests) of convergence

The most common criteria or tests of convergence include:

- Comparison test
- Ratio test
- Root test
- Integral test

Infinite number series
Dirichlet series

Dirichlet series $\sum$ is convergent if .

This series is often used in a comparison test.

Example: proove that $\sum$ is convergent.

Solution: Because each number of the given series is smaller than corresponding number of the series $\sum$.

However, this new series is convergent according to Dirichlet test. Hence, the given series is also convergent.

Infinite number series
Ratio test

This test is used in a case when a series includes a factorial.
Example: Decide whether the series $\sum$ is convergent.

$$L = \frac{\frac{2^{+}}{n}}{n!} = \frac{2}{+}$$

Conclusion: convergent.

A decision table:
If L < 1 … convergent, if L > 1 … divirgent, L= 1 … cannot be decided.

Infinite number series
Root test

The root test is used when the series includes exponential factor.

Example: Decide, whether the series $\sum$ is convergent.

Solution: $L =$

Because L < 1, the series is convergent.

Infinite number series
Root test

The root test is used when the series includes exponential factor.

Example: Decide, whether the series $\sum$ is convergent.

Solution: $L = \lim \sqrt{(\;)}$

Because L > 1, the series is divergent.

Infinite number series
Integral test

The integral test can be considered the most universal. It is usually used when other tests fail.

Example: Decide, whether the harmonic series is convergent.

Solution: $\int \quad] \quad - = +\infty$

Because the integral is infinite, also the original series is infinite, so the series is divergent.

Infinite number series
Leibniz test

Up to date, we focused on series with positive elements.

Some series might be alternating: changing plus and minus Sings from element to element.

For alternating series we can use Leibniz test:

Alternating series is convergent if $\lim_{n \to \infty} a_n =$ and $a_{n+1} < \quad \forall \in$

Infinite number series
Leibniz test - Example

Example: Decide, whether the series $\sum$ is convergent.

Solution: We chceck whether both conditions are fullfilled:

$$\lim_{n\to\infty} \frac{1}{n} = \quad \frac{1}{n+1} \quad \in$$

Both conditions are met, hence, the series is convergent.

However, the series is not *absolutely convergent*.

Infinite number series
Operations with series

Convergent series might be added or multiplied.

Operations on convergence series include:

$\sum \quad \sum$

$\sum \quad \sum \quad \sum$

Also, adding or removing a finite number of elements does not change convergence.

Infinite number series
Problems to solve - 1

Decide, whether the series is convergent:

$$\sum$$

1

$$\sum$$

$$\sum$$

$$\sum$$

$$\sum$$

Infinite number series
Problems to solve - 2

Decide, whether the series is convergent:

$$\sum$$

$$\sum$$

$$\sum$$

$$\sum$$

Infinite number series
Problems to solve - 2

Decide, whether the series is convergent:

$\sum$

$\sum$

$\sum$

$\sum$

What is next?

Next time, we will address the geometric series
and their application in economics.

The last two topics are infinite function series and differential equations.

The last lecture is devoted to problem solving.

Thank you for your attention