FV and PV of Annuity & Annuity Payment

Lecture for Corporate Finance

SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

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Outline of the lecture

- Annuity vs. Annuity payment
- PV or FV?
- Annuity and bonds
- Annuity with multi-period compounding

Annuity

An annuity is described as a stream of fixed cash flows, i.e. payments or receipts, that occurs periodically, over time.

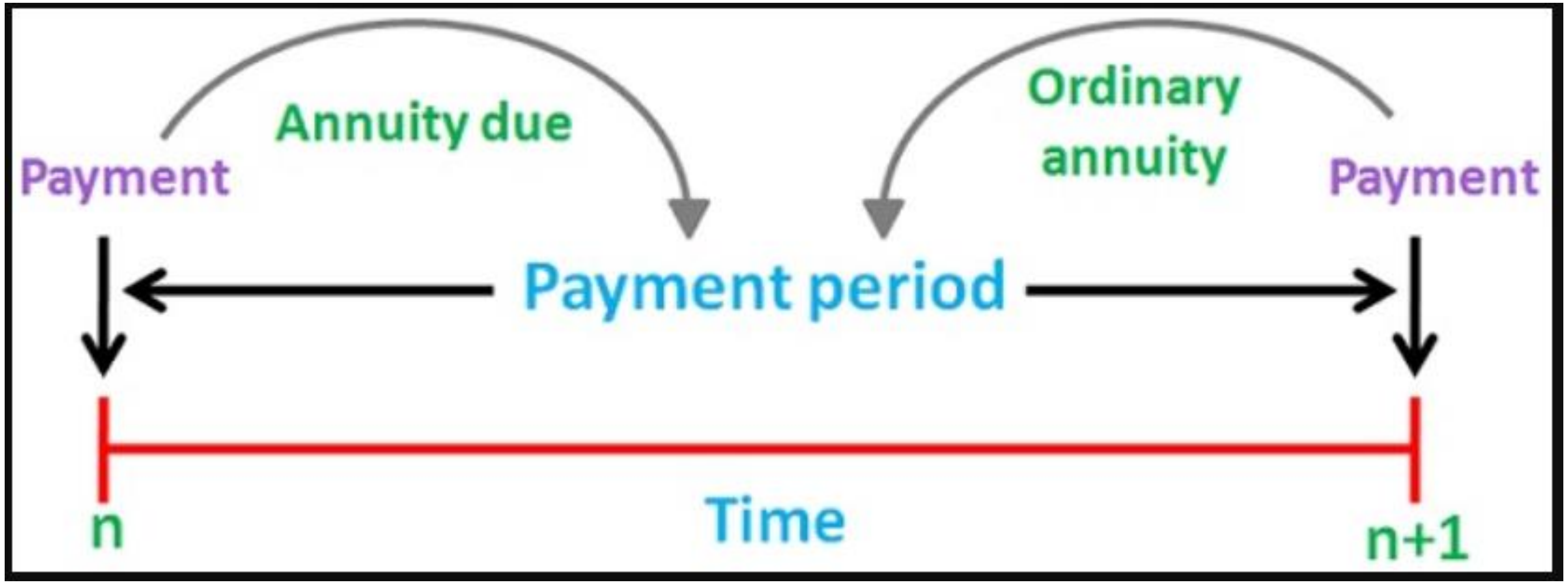
For example, payment of housing loan, life insurance premium, rent, etc. There can be two types of annuities, i.e. **ordinary annuity** and **annuity due**.

Ordinary annuity means an annuity which is related to the period preceding its date, whereas annuity due is the annuity related to the period following its date.

Ordinary Annuity is defined as a series of regular payments or receipts; that occurs at regular intervals over a specified number of periods. It is also known as annuity regular or deferred annuity.

In general, ordinary annuity payment is made on a monthly, quarterly, semi-annual or annual basis. Payment of car loan, payment of mortgage and coupon bearing bonds are some examples of an ordinary annuity. On the other hand, the common examples of an annuity due are rental lease payments, car payments, payment of life insurance premium etc.

Annuity due vs. Ordinary annuity



Annuity due vs. Ordinary annuity

Annuity due vs. Ordinary annuity

Annuity due payments are necessary to have the annuity in the future, whereas if we do need ordinary annuity now, we have to pay then annuity payments in the future.

FIRST EXAMPLE

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Ex.1 deferred annuity:

What is **the present value** of the receivable if the debtor will pay you 15 thousand CZK each year in the years 2021 - 2029. The alternative cost is 6%.

PV of an ordinary annuity

$$PV = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$PV_A = 15,000 * \frac{1.06^{9} - 1}{0.06 * 1.06^{9}}$$

- Number of ***n*** is crucial in this term ☺

Annuity payment

The annuity payment formula is used to calculate the periodic payment on an annuity. The **present value portion** of the formula is the initial payout, with an example being the original payout on an amortized loan.

The annuity payment formula can be **used for** amortized loans, income annuities, structured settlements, lottery payouts(see annuity due payment formula if first payment starts immediately), and any other type of constant periodic payments.

On the other hand, the annuity payment formula is also used to calculate the cash flows of an annuity when **future value** is known. An annuity is denoted as a series of periodic payments.

For example, if an individual is wanting **to calculate** how much they need to save per year in an interest bearing account to have a certain balance after a specific period of time, then this wanted balance would be considered the future value.

There are not only mathematical differences between calculating an annuity when present value is known and when future value is known, but also differences in the real life application.

SECOND EXAMPLE

Ex.2 present value of annuity amount - loan, loan, mortgage

How much will you repay the next 20 years if you take a loan of 750 thousand CZK? The loan bears 12% p.a.

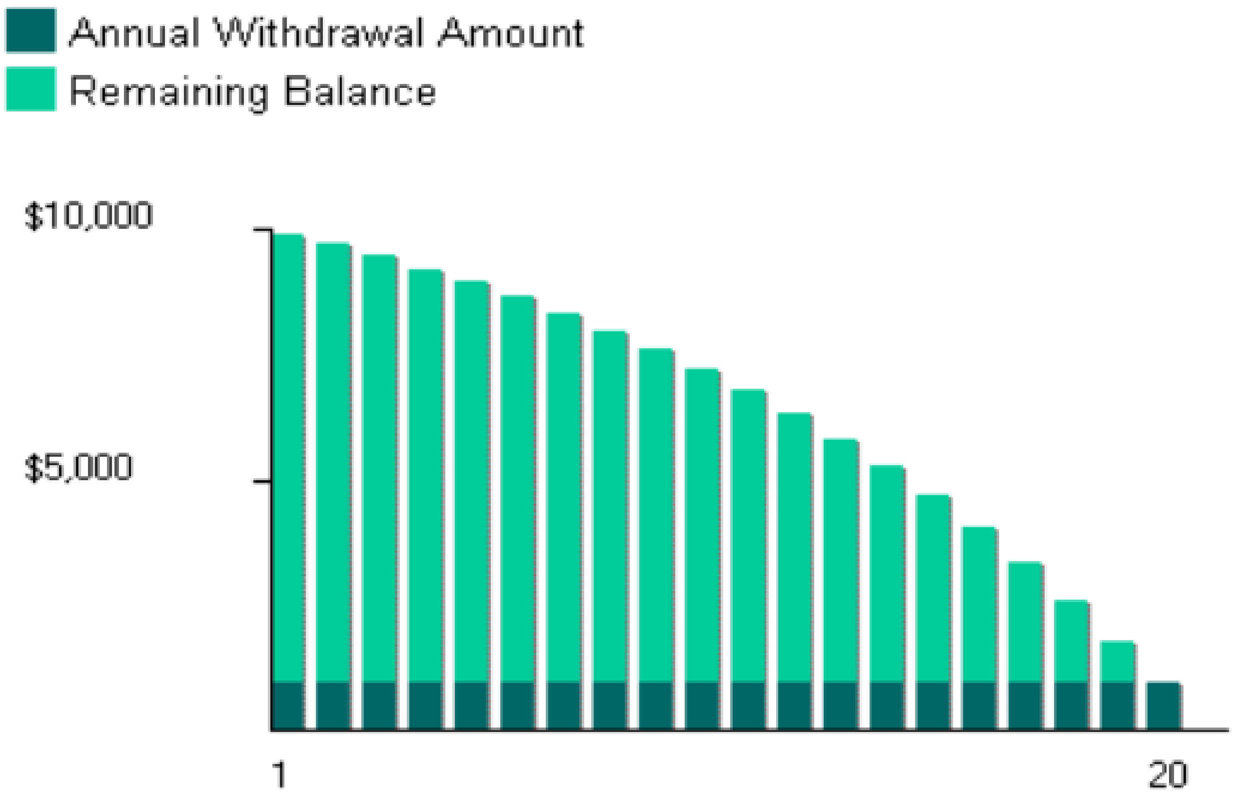
Annuity from PV

$$A = PV \frac{(1+i)^n i}{(1+i)^n - 1}$$

$$A_{PV} = 750{,}000 * \frac{0.12 * 1.12^{20}}{1.12^{20} - 1}$$

- It is necessary to understand whether we should examine PV or A.

PV of annuity (LOAN)



PV of annuity (LOAN)

PV of annuity (LOAN)

Since the start the interests are much higher from whole amount. Therefore, the annuity is decreasing slower than during the end of payment period.

Be careful of small n!

1. Consider that at the beginning of each year you will deposit 20,000 CZK on your bank account. (2020 - 2029 included) and you intend to choose savings in 2029. What will you save if the interest rate is 6% p.a.

FV of an ordinary annuity

$$FV = A\frac{(1+i)^n - 1}{i}$$

$$FV_A = 20{,}000 * \frac{1.06^{10} - 1}{0.06}$$

- If the range is from year to year, n means just a difference PLUS one ☺

Issuing bonds

When companies need to raise money, issuing bonds is one way to do it. A bond functions as a loan between an investor and a corporation.

The investor agrees to give the corporation a specific amount of money for a specific period of time in exchange for periodic interest payments at designated intervals.

When the loan reaches its maturity date, the **investor's loan is repaid**.

Corporate bonds are debt securities issued by corporations and bought by investors. They usually have higher interest rates than government bonds and are backed by the payment ability of the company.

Issuing bonds gives companies greater freedom to operate as they see fit because it releases them from the restrictions that are often attached to bank loans. Consider, for example, that lenders often require companies to agree to a variety of limitations, such as not issuing more debt or not making corporate acquisitions until their loans are repaid in full.

BONDS

2. Calculate the market price of the bond with a nominal value of CZK 10,000, coupon of 10%. Five years to maturity, coupon payment has not been paid this year. You want a yield of 9%.

PV of an ordinary annuity

$$PV = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$PV_{BOND} = 1,000 + 1,000 * \frac{1.09^5 - 1}{0.09 * 1.09^5} + \frac{10,000}{1.09^5}$$

- Analogy = we use the formulae, but we must NOT forget how to calculate a BOND! ☺

Price vs. Yield of bonds

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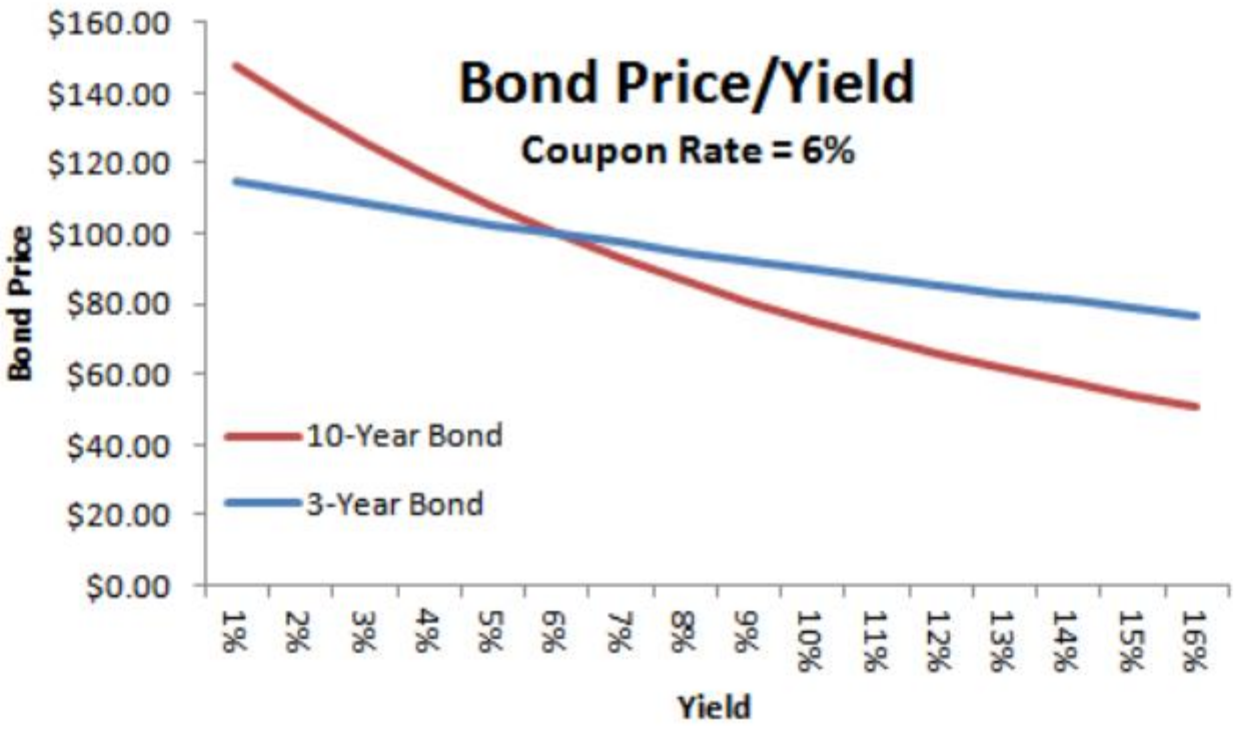

Price vs. Yield of bonds

Price vs. Yield of bonds

We can understand this yields as alternative costs. It means, if these yields on the market are smaller, it is worthy to invest to such a bond, even in the long-term.

However, when these yields would increase in the future over the fixed coupon 6%, the risk is higher and we can loose more money from our investment.

Therefore, we can argue that bonds with a longer maturity are more sensitive on the market interest rates, or yields' volatility on the market.

EAIR and combinations with annuity

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3. You will deposit CZK 1,500 each month between 2020 and 2040. The deposit bears 6% p.a. with the interest being compounded monthly. How much money will you save during that time?

Effective annual interest rate

$$EAIR = \left[1 + \frac{i}{m}\right]^m - 1$$

FV of an ordinary annuity

$$FV = A\frac{(1+i)^n - 1}{i}$$

$$EAIR = (1 + \frac{0.06}{12})^{12} - 1$$

$$FV_A = 18{,}000 * \frac{(1 + EAIR)^{21} - 1}{EAIR}$$

- If multi-period compounding is a case, always explore EAIR in the first step! ☺

BONDS = a common case of annuity

4. Calculate the market value of the bond if nine years remain to maturity, the annual coupon payment has not been paid this year and is 12%. The nominal value of the bond is CZK 10,000. Alternative costs are 8% p.a.

PV of an ordinary annuity

$$PV = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$PV_{BOND} = \mathbf{1,200} + 1{,}200 * \frac{1.08^9 - 1}{0.08 * 1.08^9} + \frac{\mathbf{10,000}}{\mathbf{1.08^9}}$$

- Do NOT forget on the coupon from previous year.

Student’s loans?

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5. In order to continue your studies at the university, you are forced to take a student loan of 70 thousand CZK interest 11% p.a. Calculate the amount of the installment if you repay once a year for 5 years.

Annuity from PV

$$A = PV \frac{(1+i)^n i}{(1+i)^n - 1}$$

$$A_{PV} = 70{,}000 * \frac{1.11^{5} * 0.11}{1.11^{5} - 1}$$

- Loan is an usual PV, cause You need the money right now ☺

Business case

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6. What is the present value of the receivable, the debtor will pay you 100 thousand CZK for six years? The debtor will start paying after four years. Your money price is 10%.

PV of an ordinary annuity

$$PV = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$a)\ PV_A = 100{,}000 * \frac{1.10^6 - 1}{0.10 * 1.10^6}$$

$$b)\ PV = \frac{PV_A}{1.10^4}$$

- A classic one is combining two sub-periods ☺

Training 1/5

7. How much will Mr. Zhang have saved in 2040 if he starts to deposit CZK 12,000 at the beginning of each year, at an annual interest rate of 4%, since 2021?

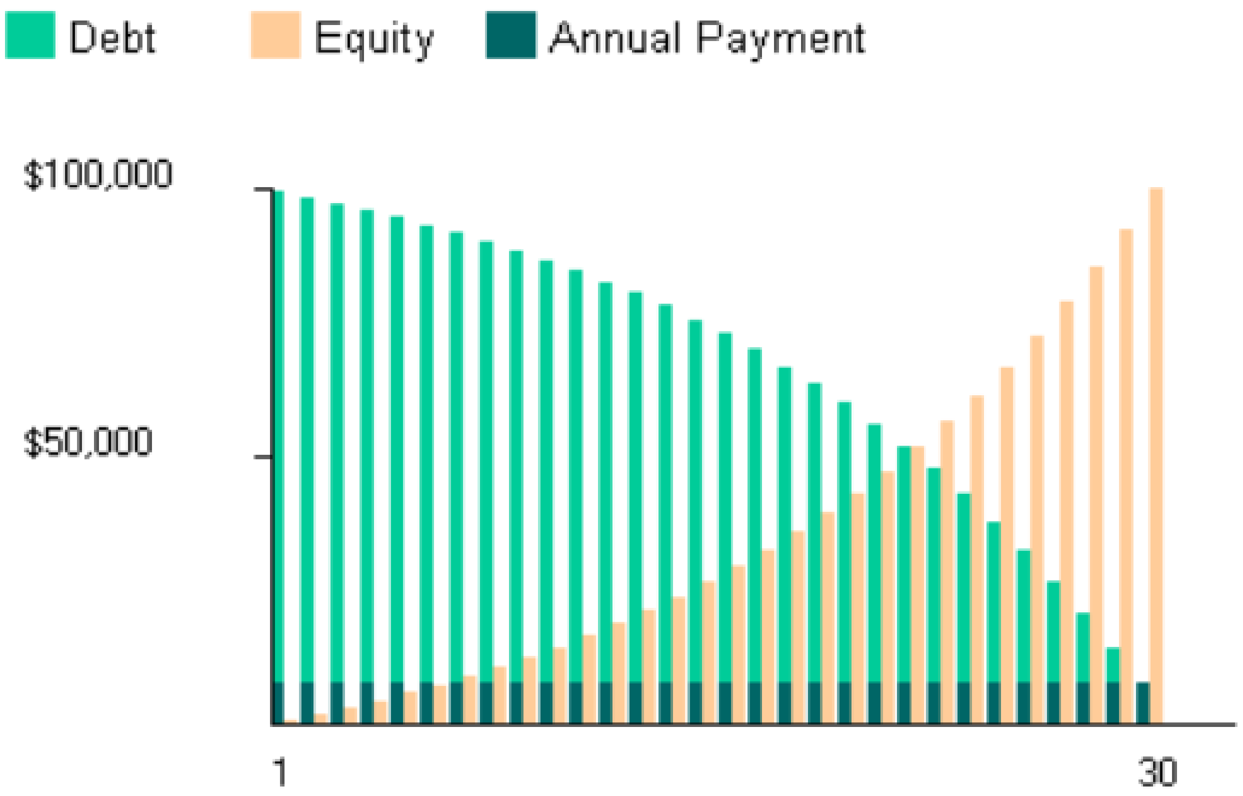
FV of an ordinary annuity

$$FV = A\frac{(1+i)^n - 1}{i}$$

$$FV_A = 12{,}000 * \frac{1.04^{20} - 1}{1{,}04}$$

- Carefully, small n! ☺

PV vs. FV of annuity



PV vs. FV of annuity

PV vs. FV of annuity

In developed economies people often combines a loan with other kind of savings, i.e. investments into the equity of companies listed on the stock exchanges.

Training 2/5

8. How much would Mr. Zhang have to save on a yearly basis to save CZK 1 million at a rate of 3.5% between 2020 and 2040?

Annuity from FV

$$A = FV \frac{i}{(1+i)^n - 1}$$

$$A_{FV} = 1{,}000{,}000 * \frac{0.035}{1.035^{21} - 1}$$

- What do we know in this example? *FV*

Training 3/5

9. Mrs. Zhang has CZK 500,000 saved, of which she intends to draw a pension every year for 12 years. How high can an annual income be expected if the interest rate is 8%?

Annuity from PV

$$A = PV \frac{(1+i)^n i}{(1+i)^n - 1}$$

$$A_{PV} = 500{,}000 * \frac{0.08 * 1.08^{12}}{1.08^{12} - 1}$$

- Please, always remember a possible multi-period compounding ☺ (then, EAIR as the first step!)

Training 4/5

10. You want to take a mortgage. How high would a 25-year loan be if you were able to repay a maximum of CZK 60,000 per year? The interest rate is 10%.

PV of an ordinary annuity

$$PV = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$PV_A = 60{,}000 * \frac{1.10^{25} - 1}{0.10 * 1.10^{25}}$$

- If You are buying a house/flat, You usually take a mortgage, loan.

Training 5/5

11. What is the present value of a bond with a maturity of 15 years, with a nominal value of 400 thousand CZK, with a coupon payment of 5%, which you intend to sell for 420 thousand CZK after 10 years? You require a yield of 7% p. a.

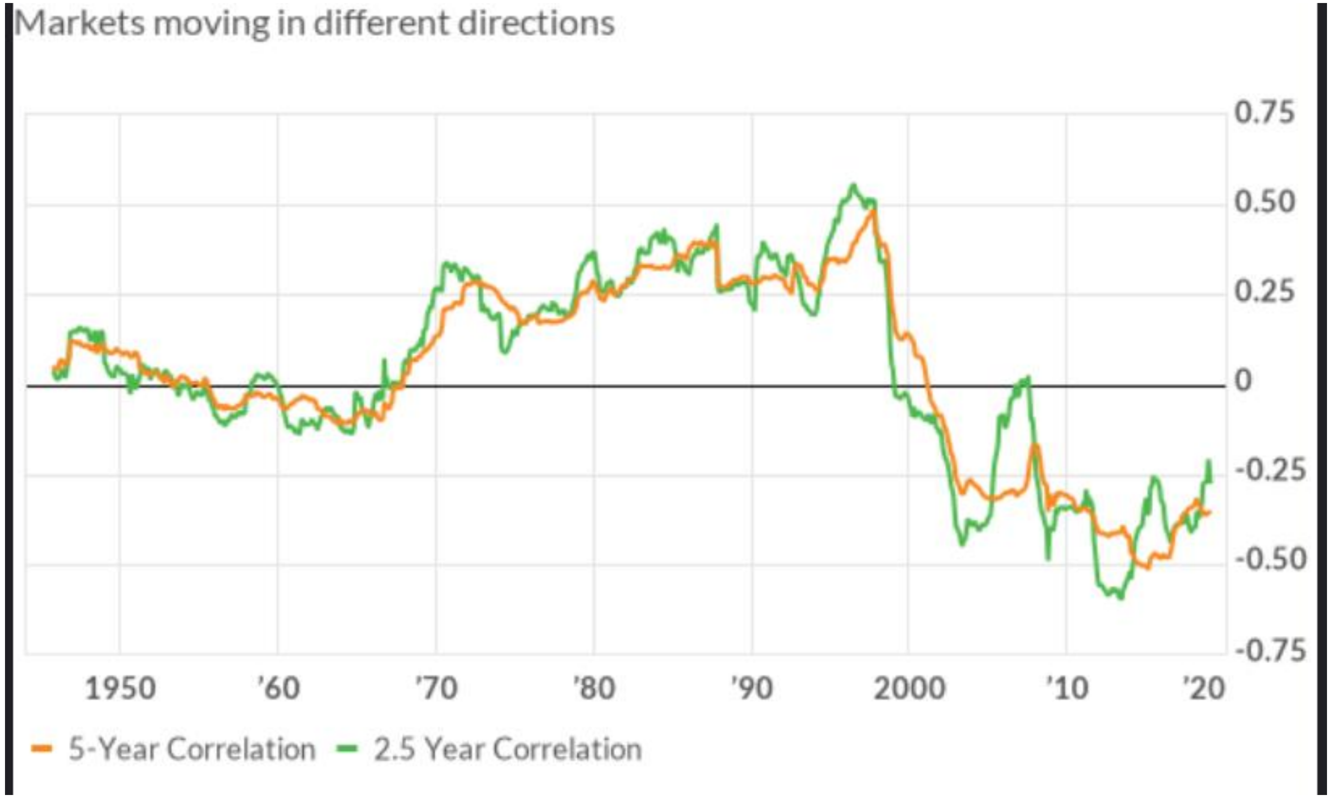
PV of an ordinary annuity

$$PV = A\frac{(1+i)^n - 1}{i(1+i)^n}$$

$$PV_{BOND} = 20{,}000 * \frac{1.07^{10} - 1}{0.07 * 1.07^{10}} + \frac{420{,}000}{1.07^{10}}$$

- You will sell this bond earlier than it expires.

Correlation stock and bonds prices



Correlation stock and bonds prices

We can see that development of stock and bond prices is negative in the long-term. Therefore, for hold and buy strategy could be beneficial to include even the bonds into investor's portfolio. Many companies are institutional investors as well.

Literature

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1) Seminar 04 Examples
2) ROSS, S. A., R. W. WESTERFIELD, J. JAFFE & B. D. JORDAN, 2019. Valuation and Capital Budgeting. In: Corporate Finance, PART II, pp. 85-298. ISBN 978-1-260-09187-8.
3) BERK, J. & P. DeMARZO, 2017. The Time Value of Money. In: Corporate Finance, Chap. 4, pp. 130-174. ISBN 978-1-292-16016-0.

- Next time will be one of the hardest lectures, growing annuity and perpetuity! Look forward to that ☺

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Thank you for your attention!

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