

Funkce jedné proměnné

- definice funkce
- explicitní a implicitní zápis
- definiční obor, obor funkčních hodnot, graf

Základní vlastnosti:

monotónnost funkce

funkce inverzní

složená funkce

konkávní a konvexní

funkce sudá, lichá

omezenost funkce

funkce prostá

-sup a inf funkce

Algebraické funkce

- Konstantní
- Lineární
- Kvadratická
- Mocniné (potenční)
- Druhá a třetí odmocnina

Konstatntní funkce

Je to funkce ve tvaru

$$y = c$$

Grafem je přímka rovnoběžná s osou x .

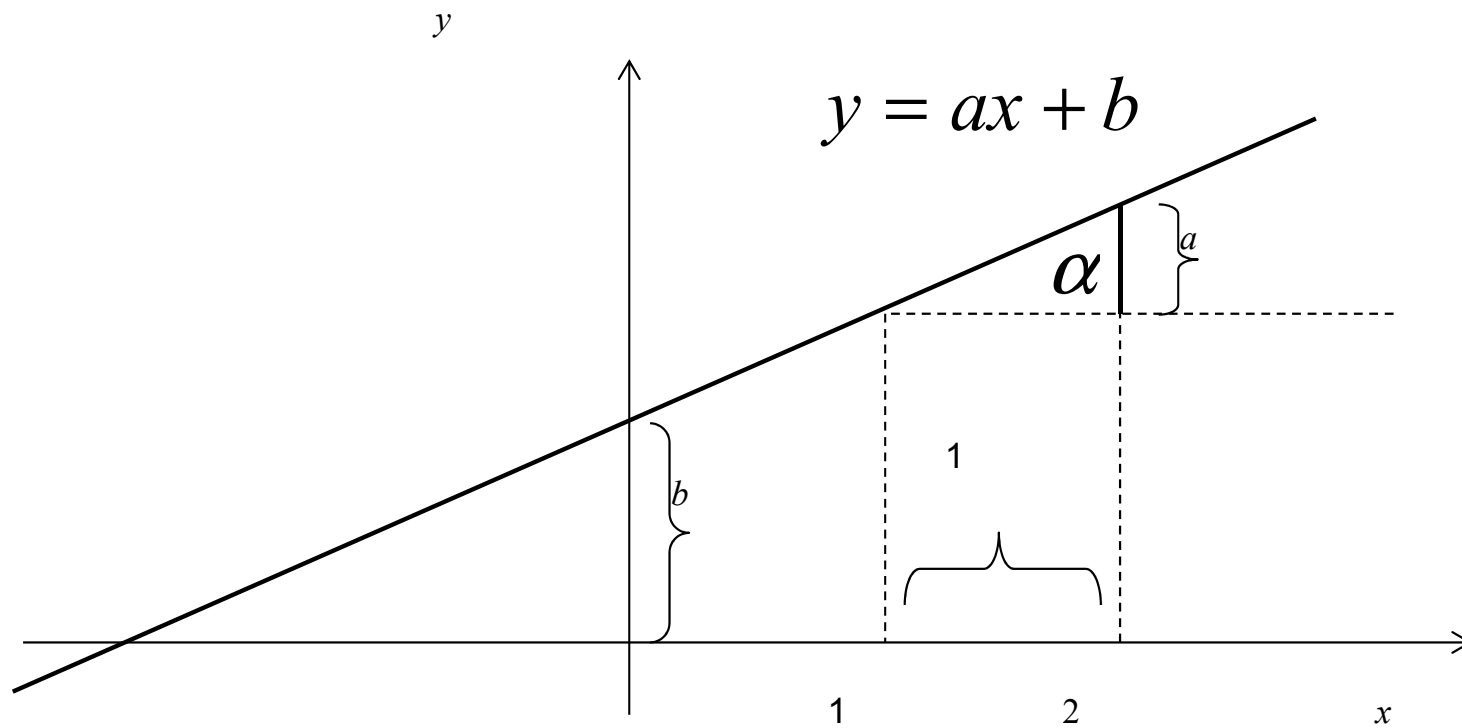
Lineární funkce je funkce ve tvaru:

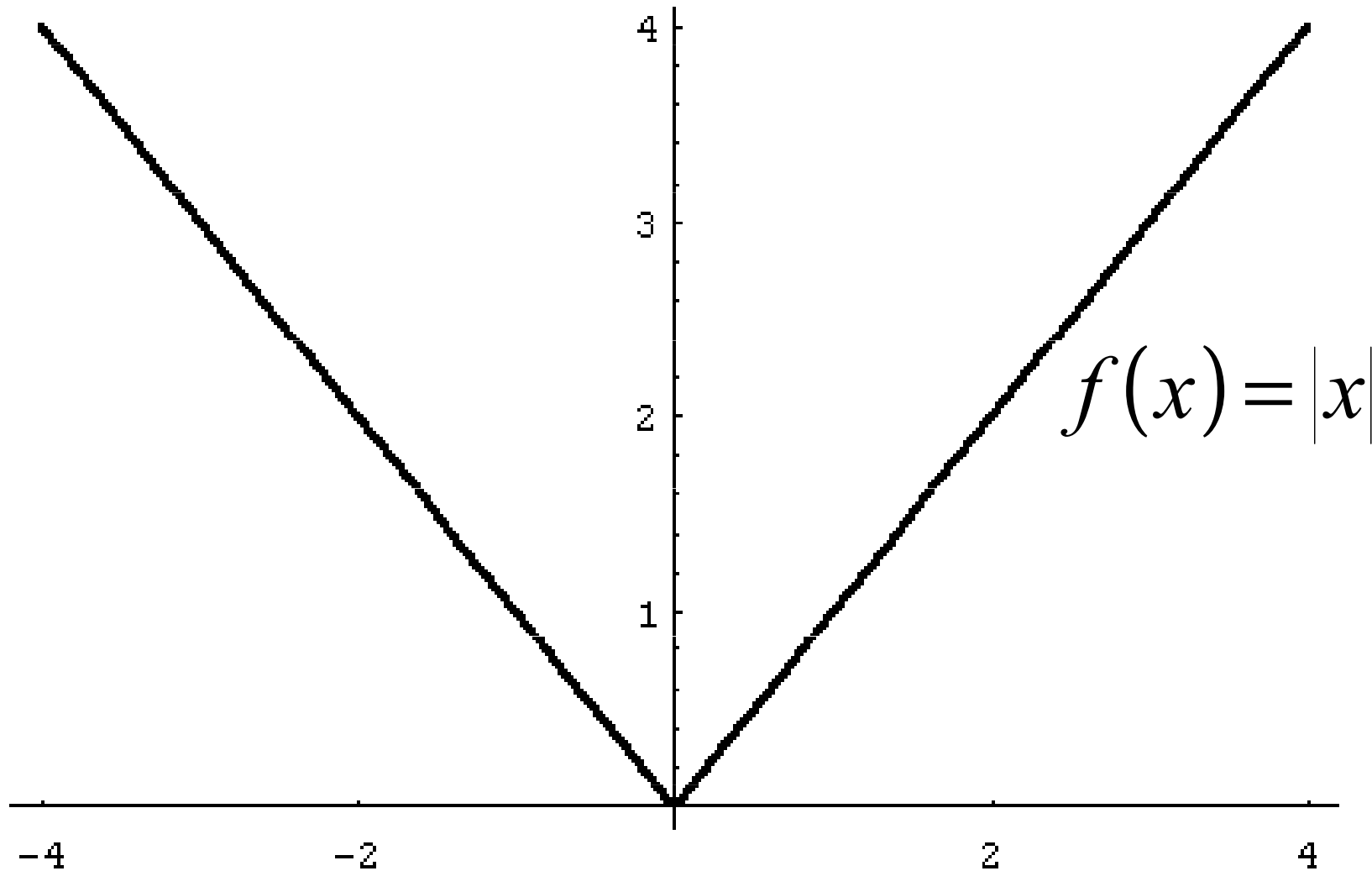
$$y = ax + b$$

Grafem této funkce je přímka. Jednotlivé koeficienty mají tento význam:

- a - směrnice přímky, která je grafem lineární funkce,
- b - úsek (vytátný přímkou) na ose y

Graf lineární funkce .

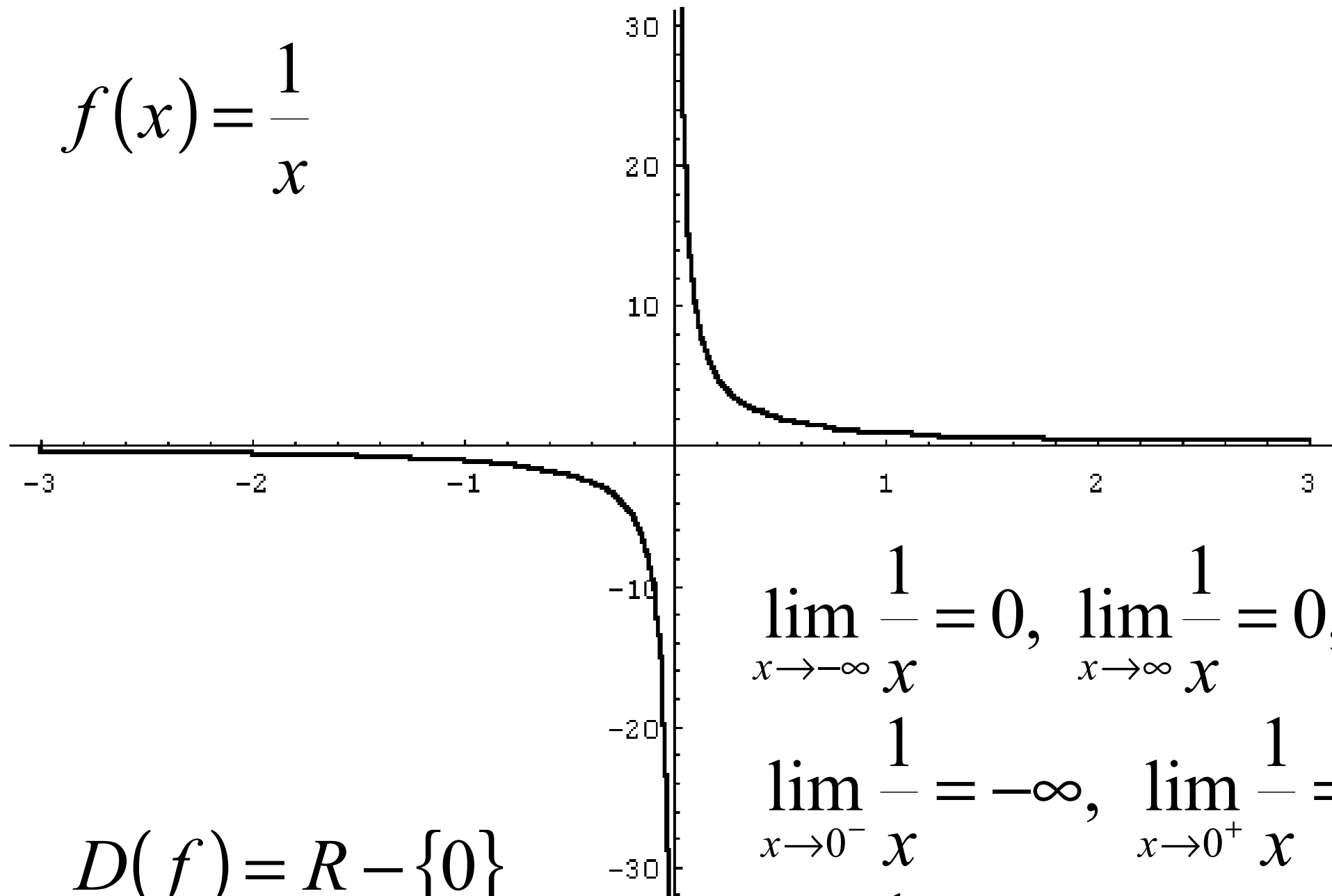




$$D(f) = R, \quad H(f) = \langle 0, \infty \rangle,$$

$$\lim_{x \rightarrow -\infty} |x| = \infty, \quad \lim_{x \rightarrow \infty} |x| = \infty$$

$$f(x) = \frac{1}{x}$$



$$D(f) = \mathbb{R} - \{0\}$$

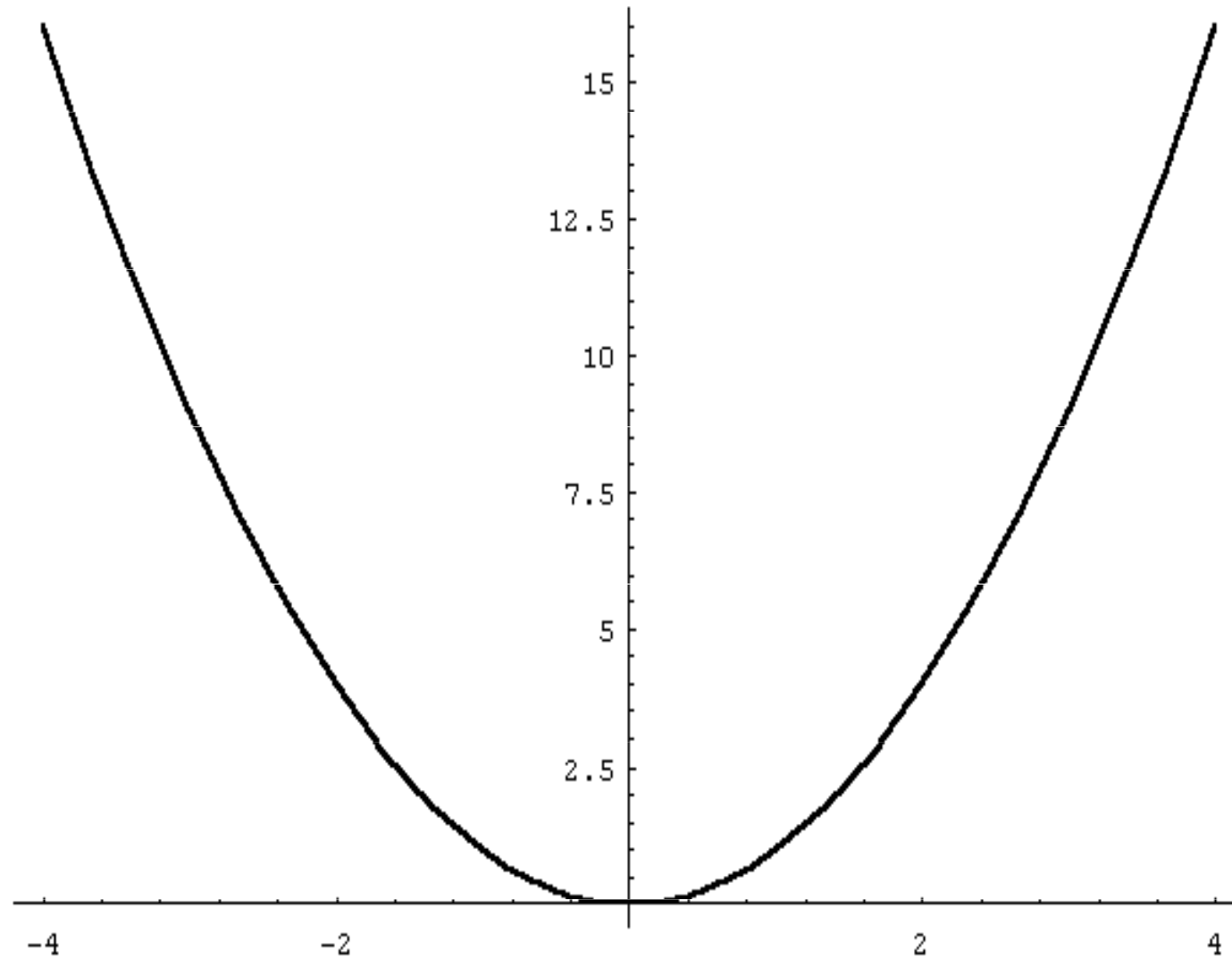
$$H(f) = \mathbb{R} - \{0\}$$

$$\lim_{x \rightarrow -\infty} \frac{1}{x} = 0, \quad \lim_{x \rightarrow \infty} \frac{1}{x} = 0,$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty, \quad \lim_{x \rightarrow 0^+} \frac{1}{x} = \infty,$$

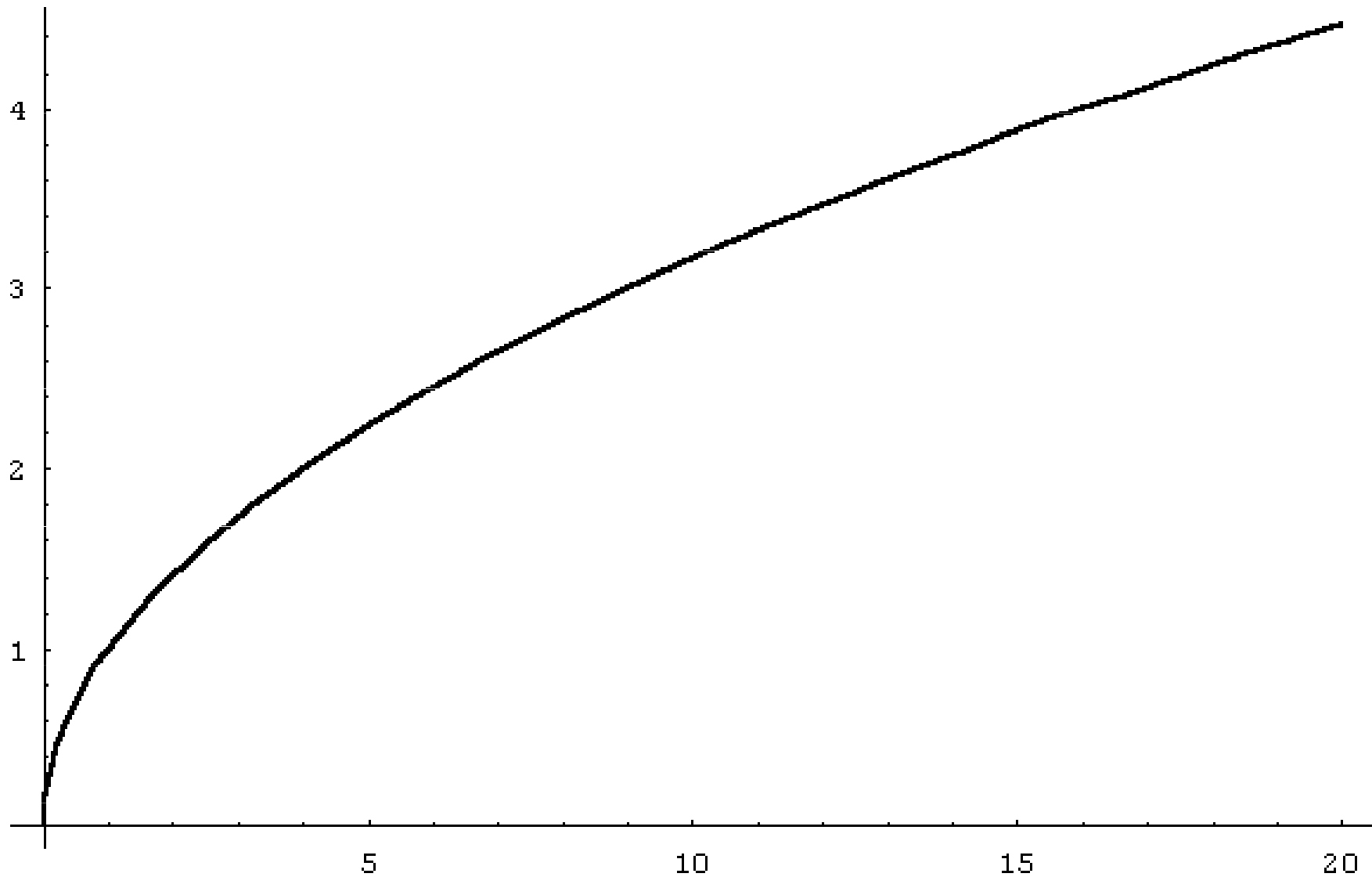
$$\lim_{x \rightarrow 0} \frac{1}{x} \quad \text{neexistuje}$$

$$f(x) = x^2$$



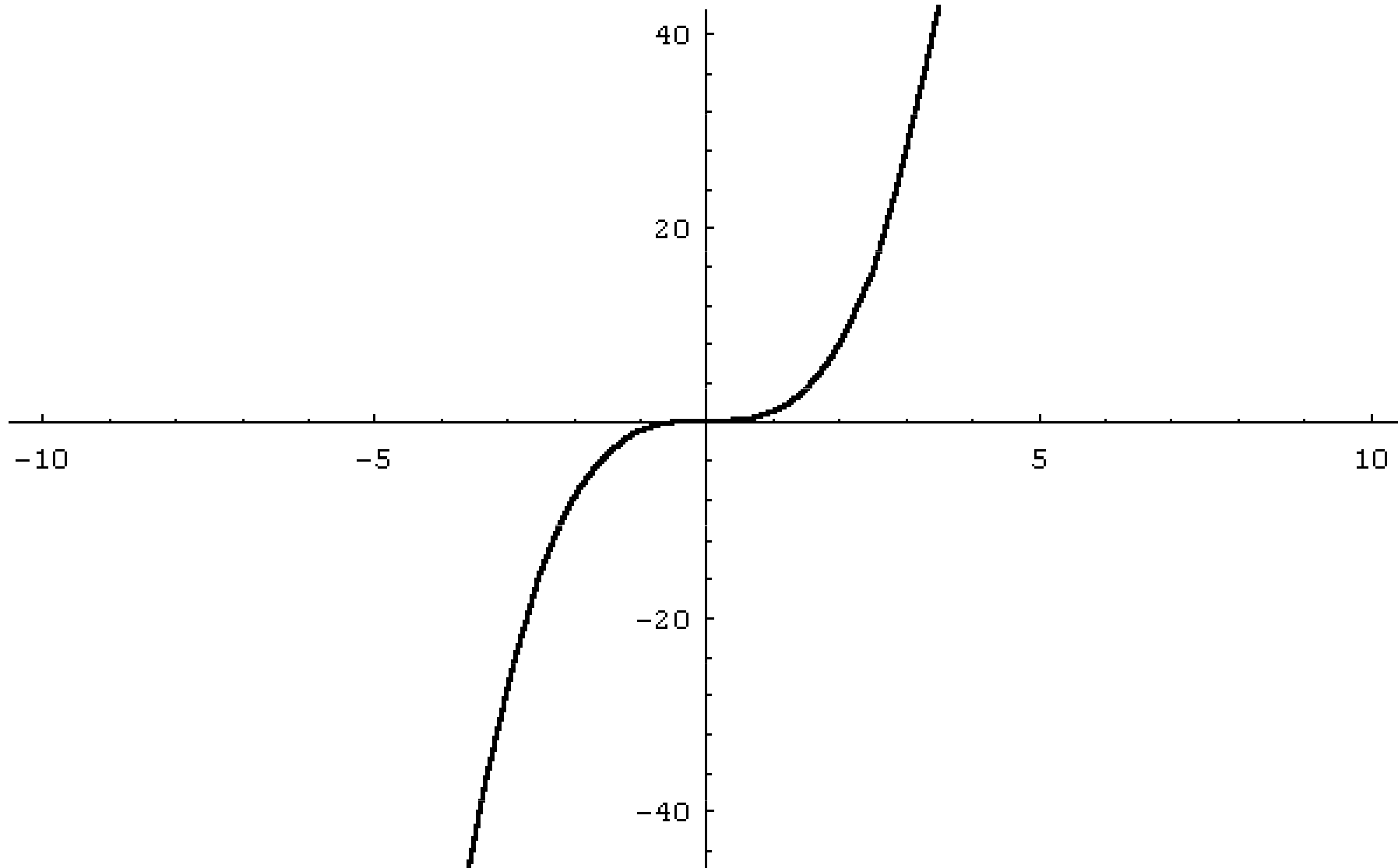
$$D(f) = \mathbb{R}, \quad H(f) = \langle 0, \infty \rangle, \quad \lim_{x \rightarrow -\infty} x^2 = \infty, \quad \lim_{x \rightarrow \infty} x^2 = \infty$$

$$f(x) = \sqrt{x}$$



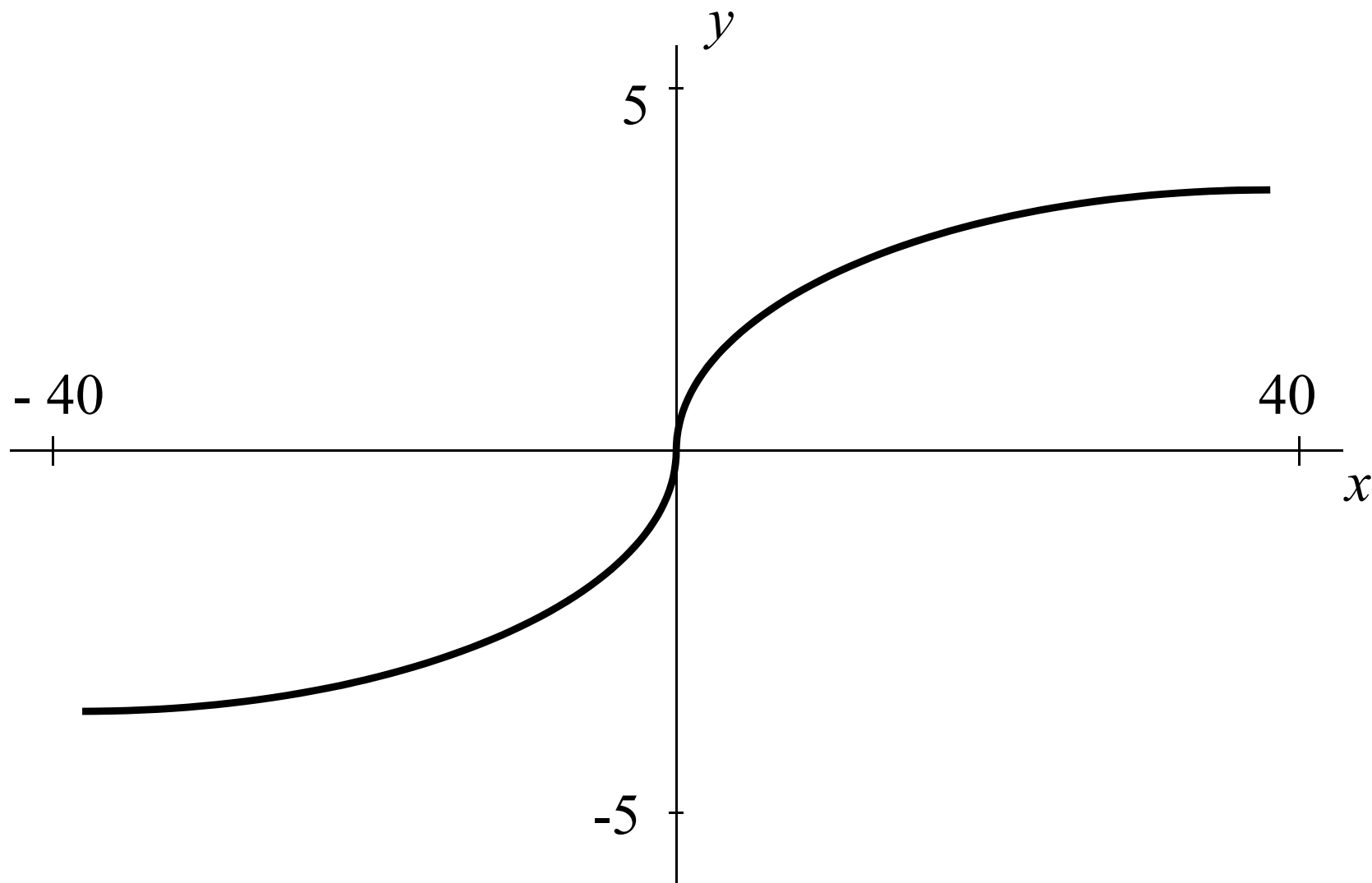
$$D(f) = \langle 0, \infty \rangle, \quad H(f) = \langle 0, \infty \rangle, \quad \lim_{x \rightarrow \infty} \sqrt{x} = \infty$$

$$f(x) = x^3$$



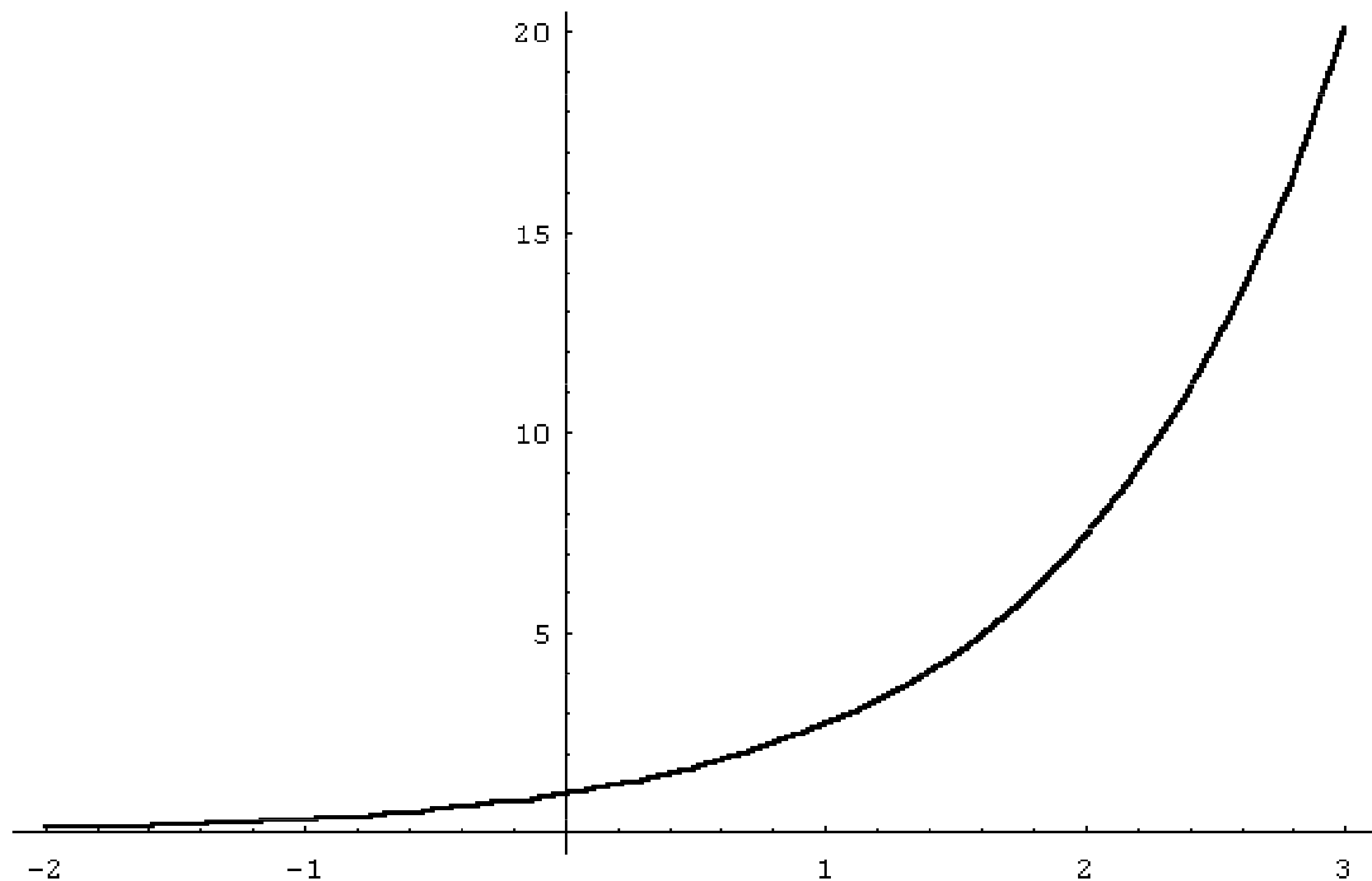
$$D(f) = R, \quad H(f) = R, \quad \lim_{x \rightarrow -\infty} x^3 = -\infty, \quad \lim_{x \rightarrow \infty} x^3 = \infty$$

$$f(x) = \sqrt[3]{x}$$



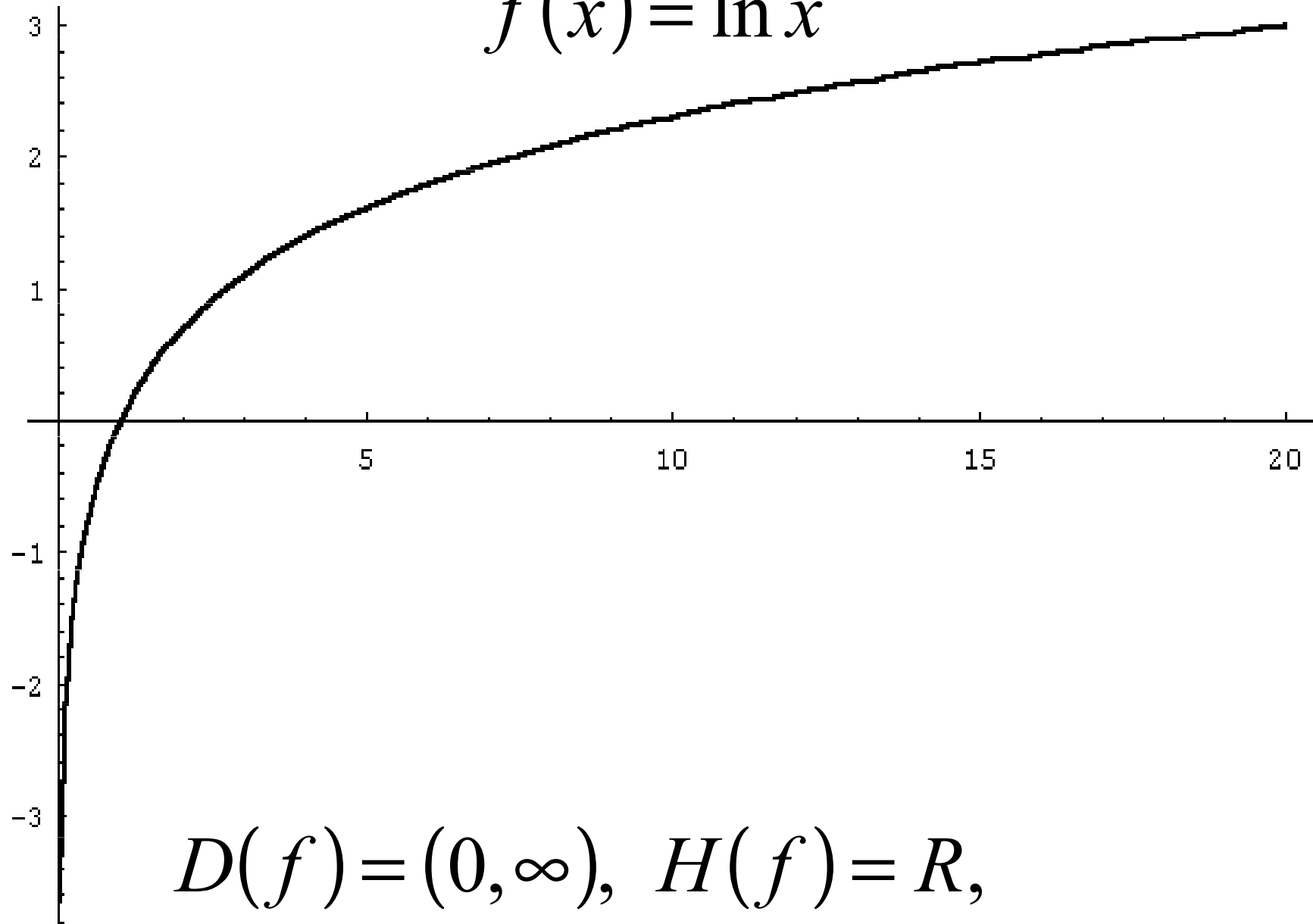
$$D(f) = R, \quad H(f) = R, \quad \lim_{x \rightarrow -\infty} \sqrt[3]{x} = -\infty, \quad \lim_{x \rightarrow \infty} \sqrt[3]{x} = \infty$$

$$f(x) = e^x$$



$$D(f) = R, \quad H(f) = (0, \infty), \quad \lim_{x \rightarrow -\infty} e^x = 0, \quad \lim_{x \rightarrow \infty} e^x = \infty$$

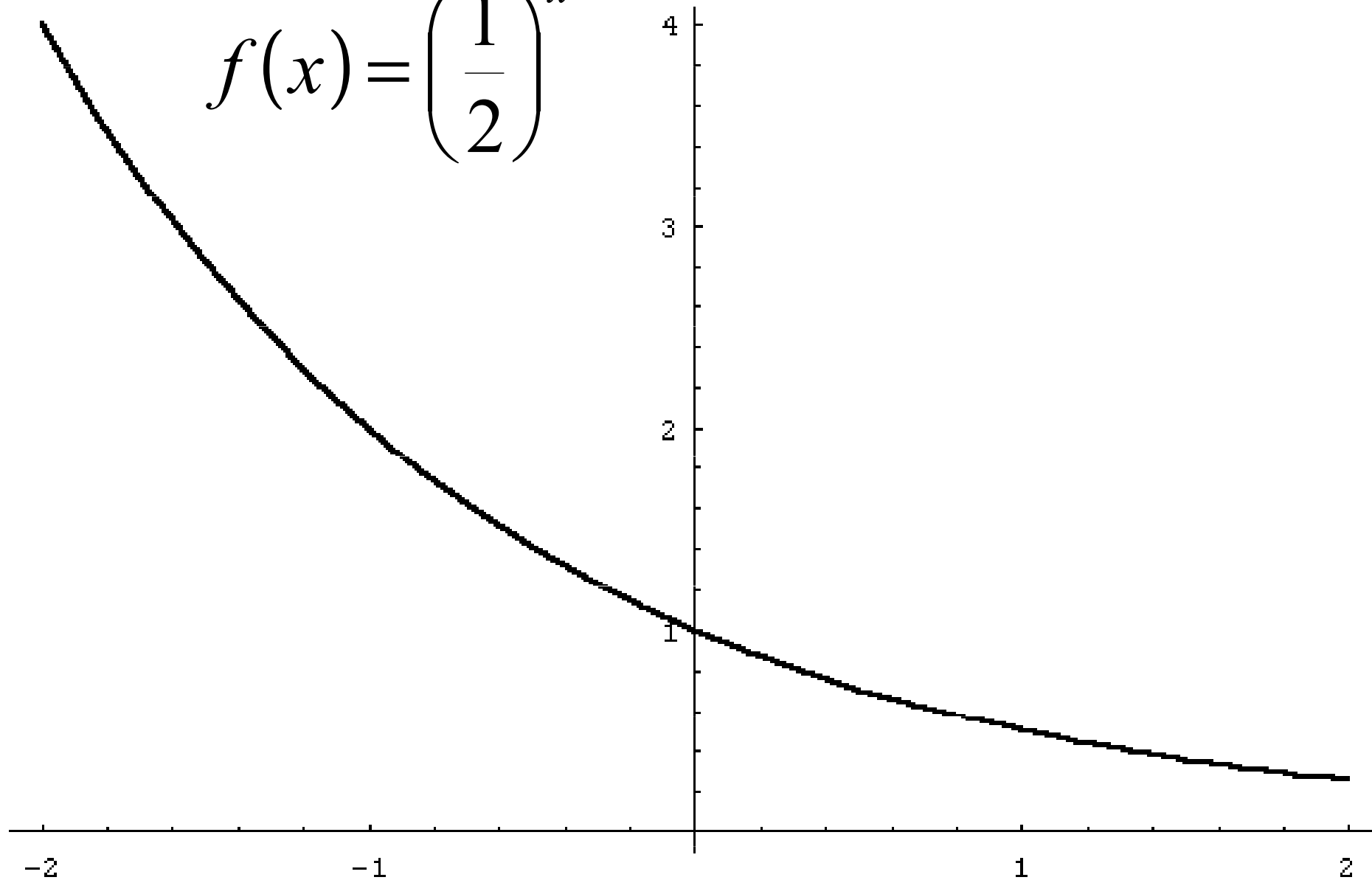
$$f(x) = \ln x$$



$$D(f) = (0, \infty), \quad H(f) = \mathbb{R},$$

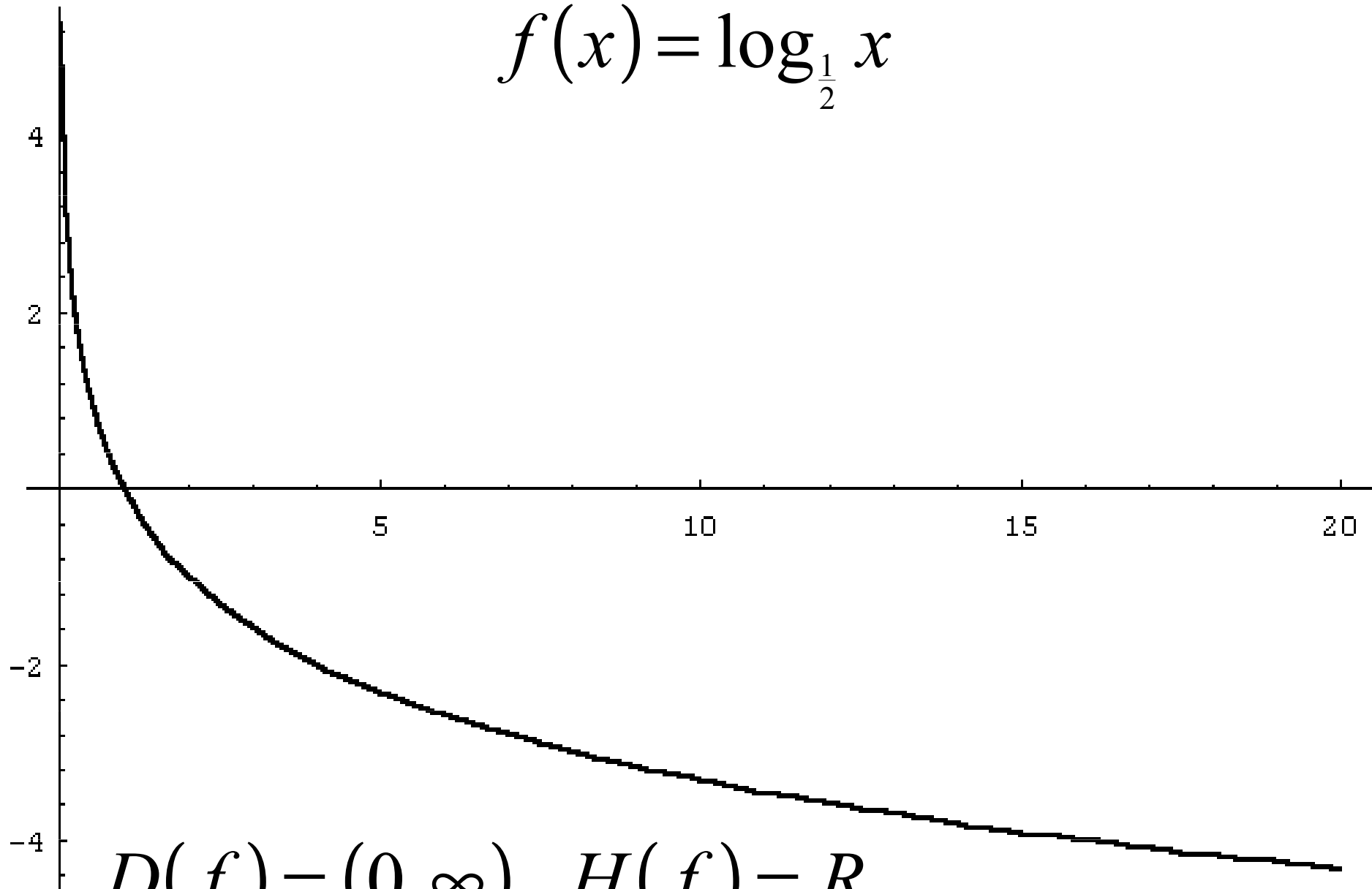
$$\lim_{x \rightarrow 0^+} \ln x = -\infty, \quad \lim_{x \rightarrow \infty} \ln x = \infty$$

$$f(x) = \left(\frac{1}{2}\right)^x$$



$$D(f) = R, \quad H(f) = (0, \infty), \quad \lim_{x \rightarrow -\infty} \left(\frac{1}{2}\right)^x = \infty, \quad \lim_{x \rightarrow \infty} \left(\frac{1}{2}\right)^x = 0$$

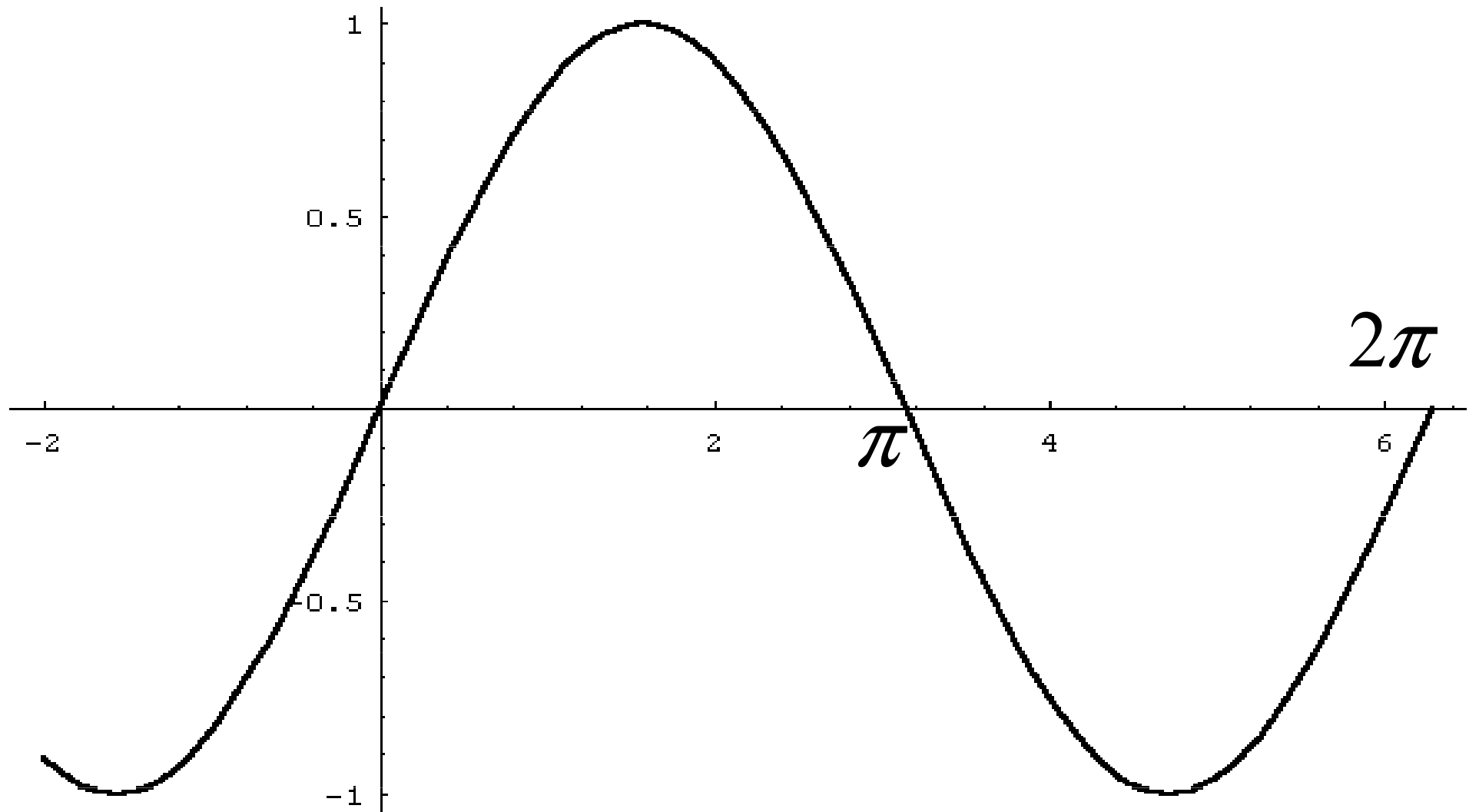
$$f(x) = \log_{\frac{1}{2}} x$$



$$D(f) = (0, \infty), \quad H(f) = R,$$

$$\lim_{x \rightarrow 0^+} \log_{\frac{1}{2}} x = \infty, \quad \lim_{x \rightarrow \infty} \log_{\frac{1}{2}} x = -\infty$$

$$f(x) = \sin x$$

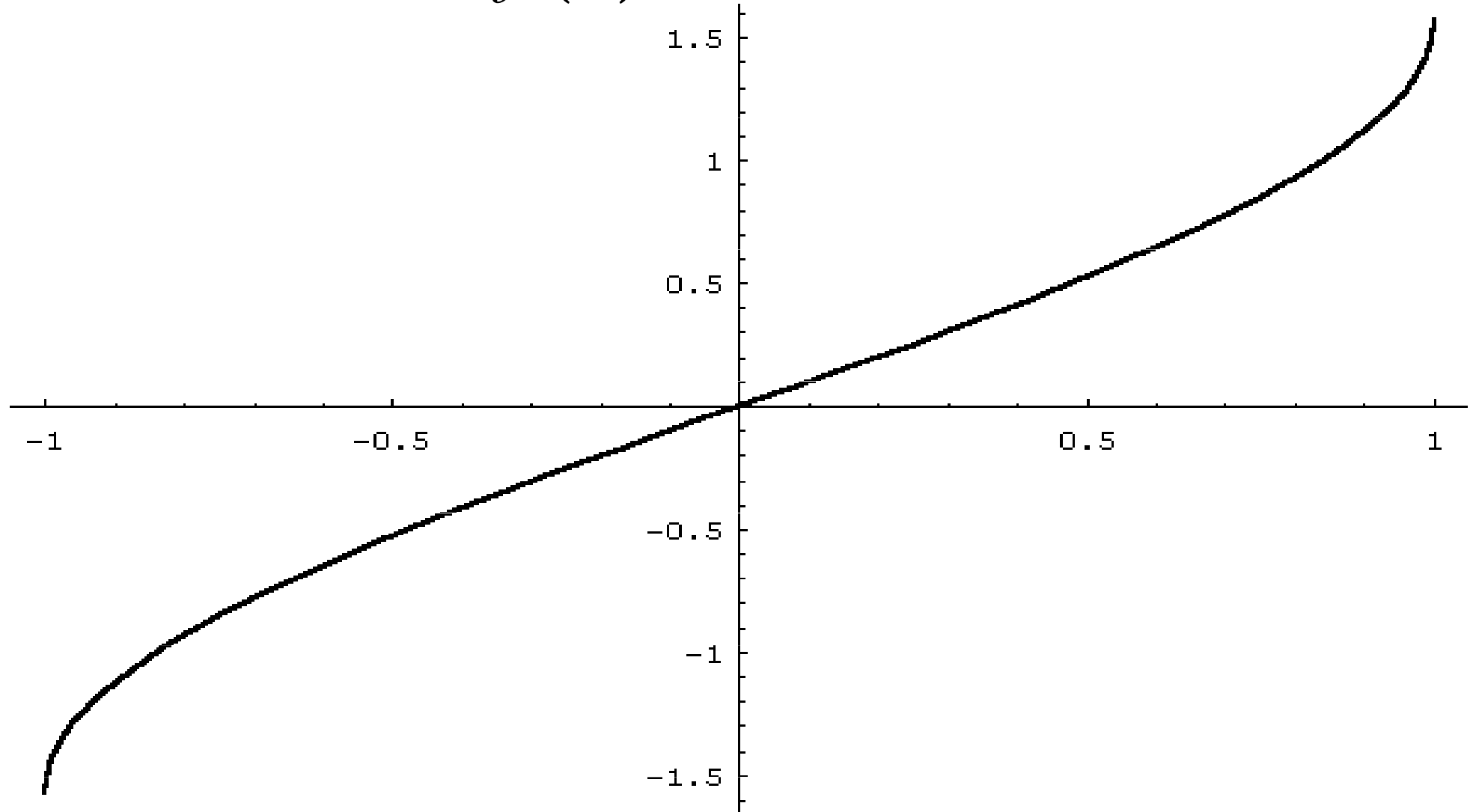


$$D(f) = R, \quad H(f) = \langle -1, 1 \rangle,$$

$\lim_{x \rightarrow \infty} \sin x$ neexist.

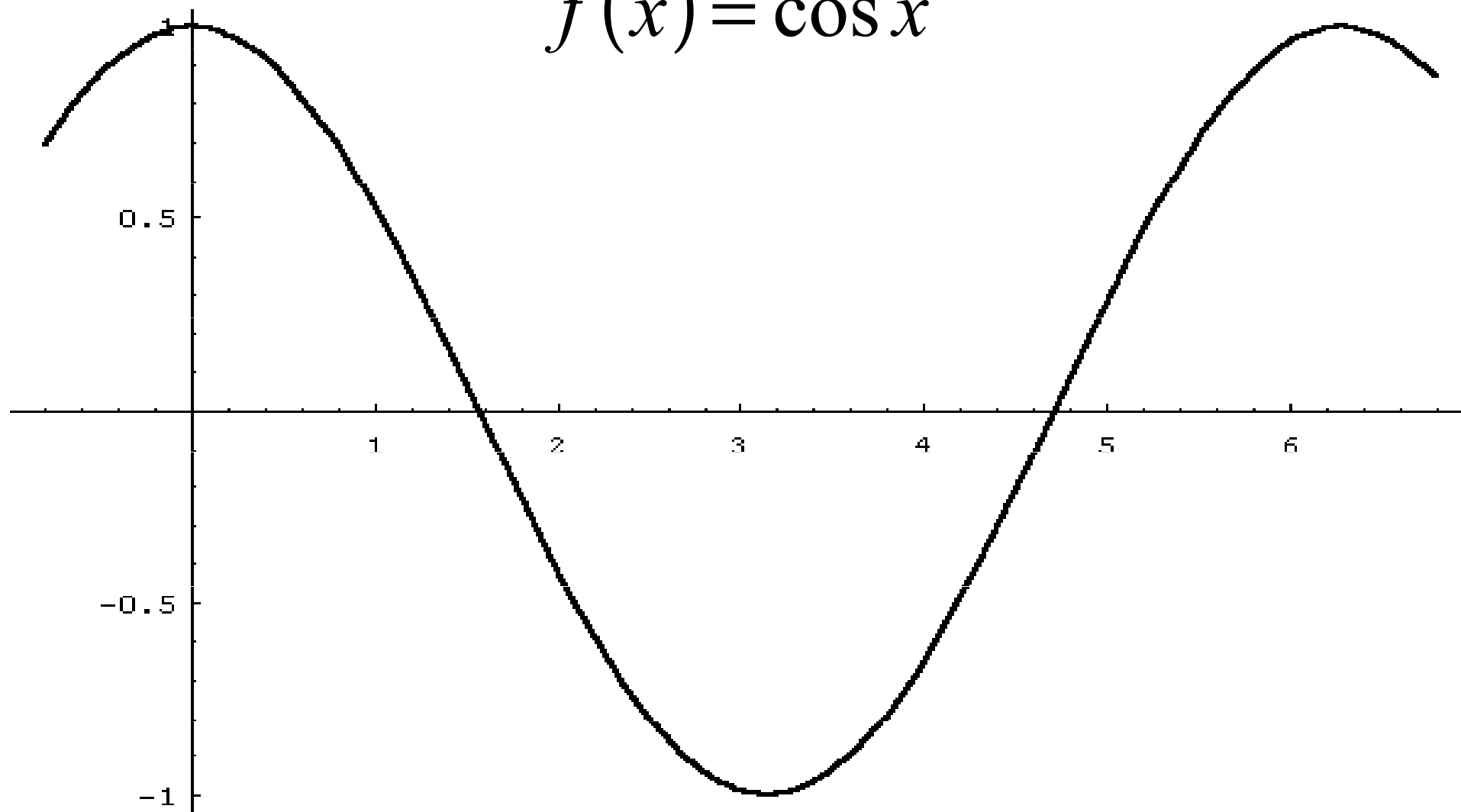
$\lim_{x \rightarrow -\infty} \sin x$ neexist.

$$f(x) = \arcsin x$$



$$D(f) = \langle -1, 1 \rangle, \quad H(f) = \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$$

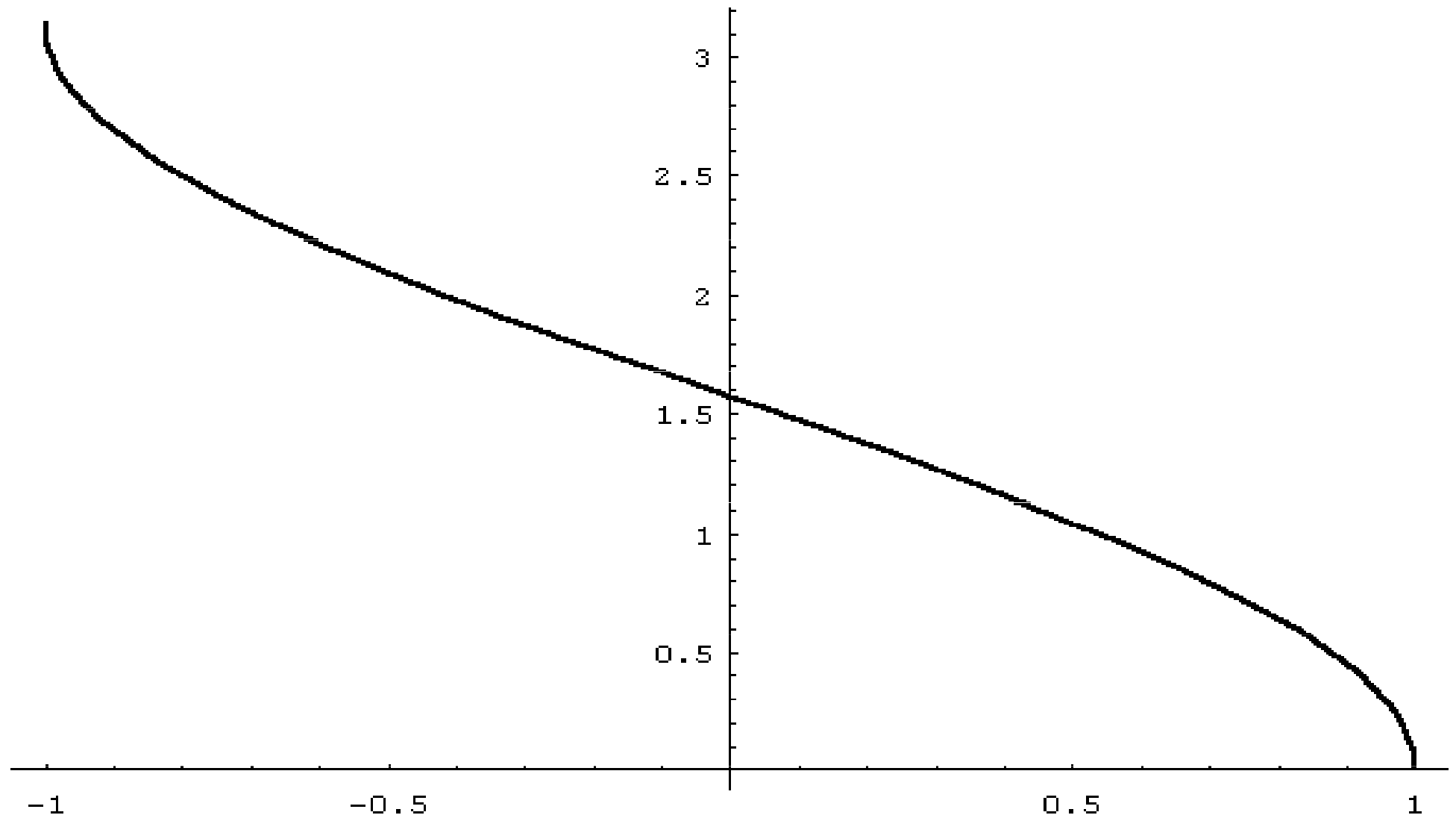
$$f(x) = \cos x$$



$$D(f) = \mathbb{R}, \quad H(f) = \langle -1, 1 \rangle,$$

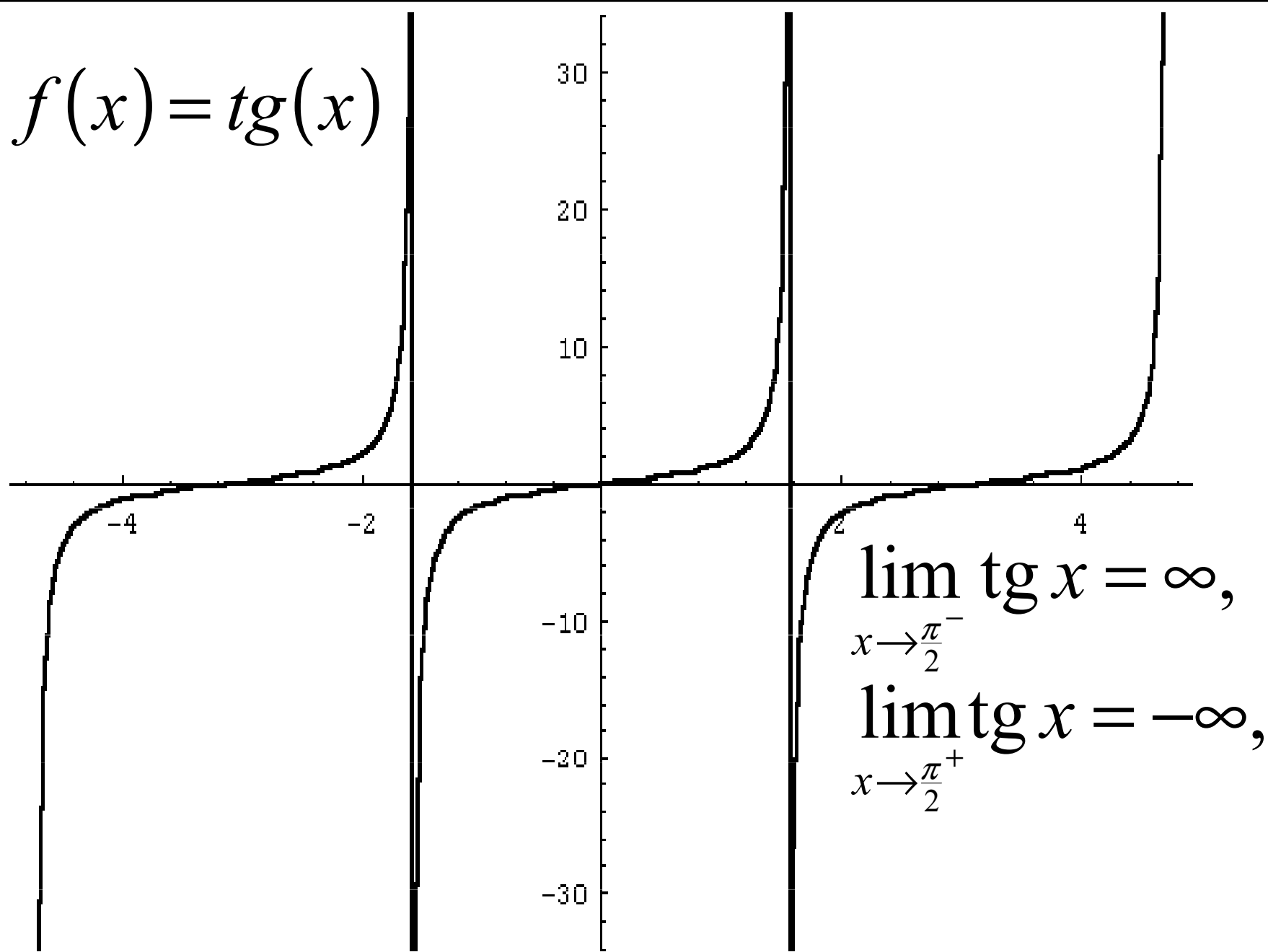
$\lim_{x \rightarrow \infty} \cos x$ neexistuje, $\lim_{x \rightarrow -\infty} \cos x$ neexistuje,

$$f(x) = \arccos x$$



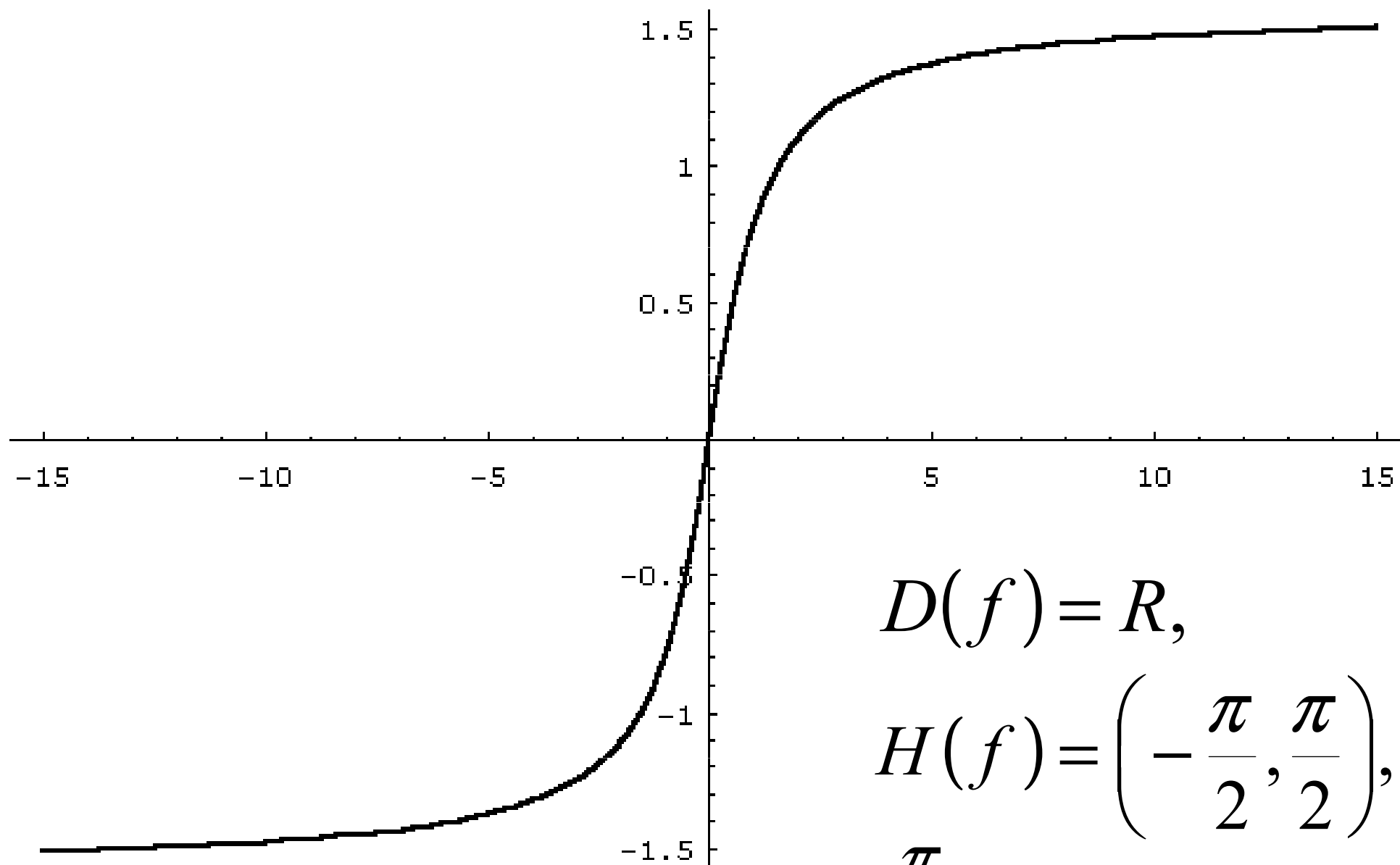
$$D(f) = \langle -1, 1 \rangle, \quad H(f) = \langle 0, \pi \rangle$$

$$f(x) = \operatorname{tg}(x)$$



$$D(f) = R - \left\{ (2k+1)\frac{\pi}{2}, k \in Z \right\}, H(f) = R$$

$$f(x) = \operatorname{arctg} x$$

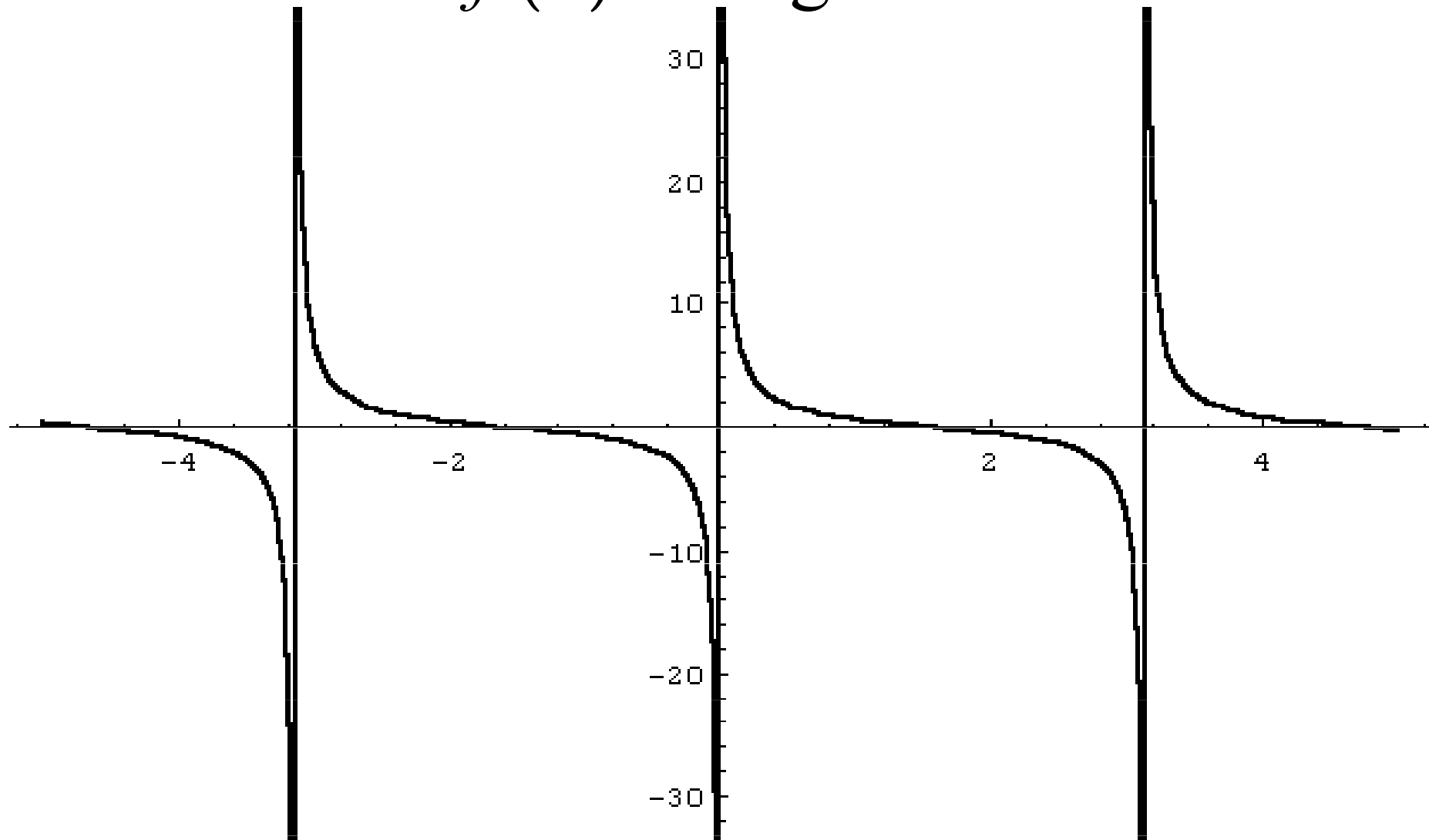


$$D(f) = R,$$

$$H(f) = \left(-\frac{\pi}{2}, \frac{\pi}{2} \right),$$

$$\lim_{x \rightarrow -\infty} \operatorname{arctg} x = -\frac{\pi}{2}, \quad \lim_{x \rightarrow \infty} \operatorname{arctg} x = \frac{\pi}{2}$$

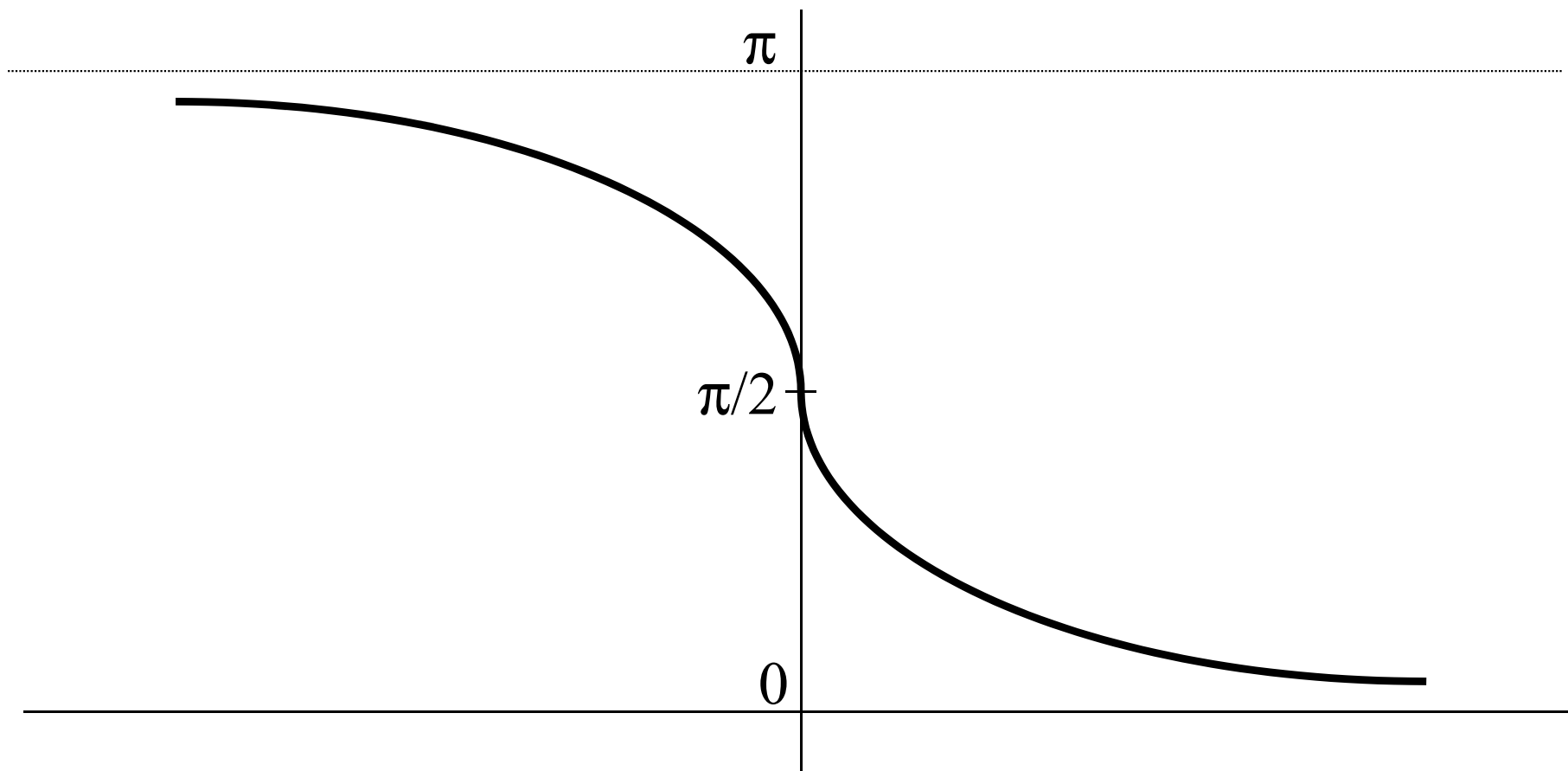
$$f(x) = \operatorname{cotg} x$$



$$D(f) = R - \{k\pi, k \in Z\}, \quad H(f) = R$$

$$\lim_{x \rightarrow 0^-} \operatorname{cotg} x = -\infty, \quad \lim_{x \rightarrow 0^+} \operatorname{cotg} x = \infty, \quad \lim_{x \rightarrow 0} \operatorname{cotg} x \text{ neexist.}$$

$$f(x) = \operatorname{arccotg} x$$



$$D(f) = R, \quad H(f) = (0, \pi),$$

$$\lim_{x \rightarrow -\infty} \operatorname{arccotg} x = \pi, \quad \lim_{x \rightarrow \infty} \operatorname{arccotg} x = 0$$

Příklad

Určete $D(f)$ funkce

$$y = \arcsin(2x - 1) + \frac{3e^{1/x}}{\sqrt{x^2 - 3x + 2}}$$