Funkce jedné proměnné

- definice funkce
- explicitní a implicitní zápis
- definiční obor, obor funkčních hodnot, graf

Základní vlastnosti:

- monotónnost funkce
- složená funkce
- funkce sudá, lichá
- funkce prostá
- funkce inverzní
- konkávní a konvexní
- omezenost funkce

- sup a inf funkce

Algebraické funkce

- Konstantní
- Lineární
- Kvadratická
- Mocnniné (potenční)
- Druhá a třetí odmocnina

Konstatntní funkce

Je to funkce ve tvaru

$$y = c$$

Grafem je přímka rovnoběžná s osou x.

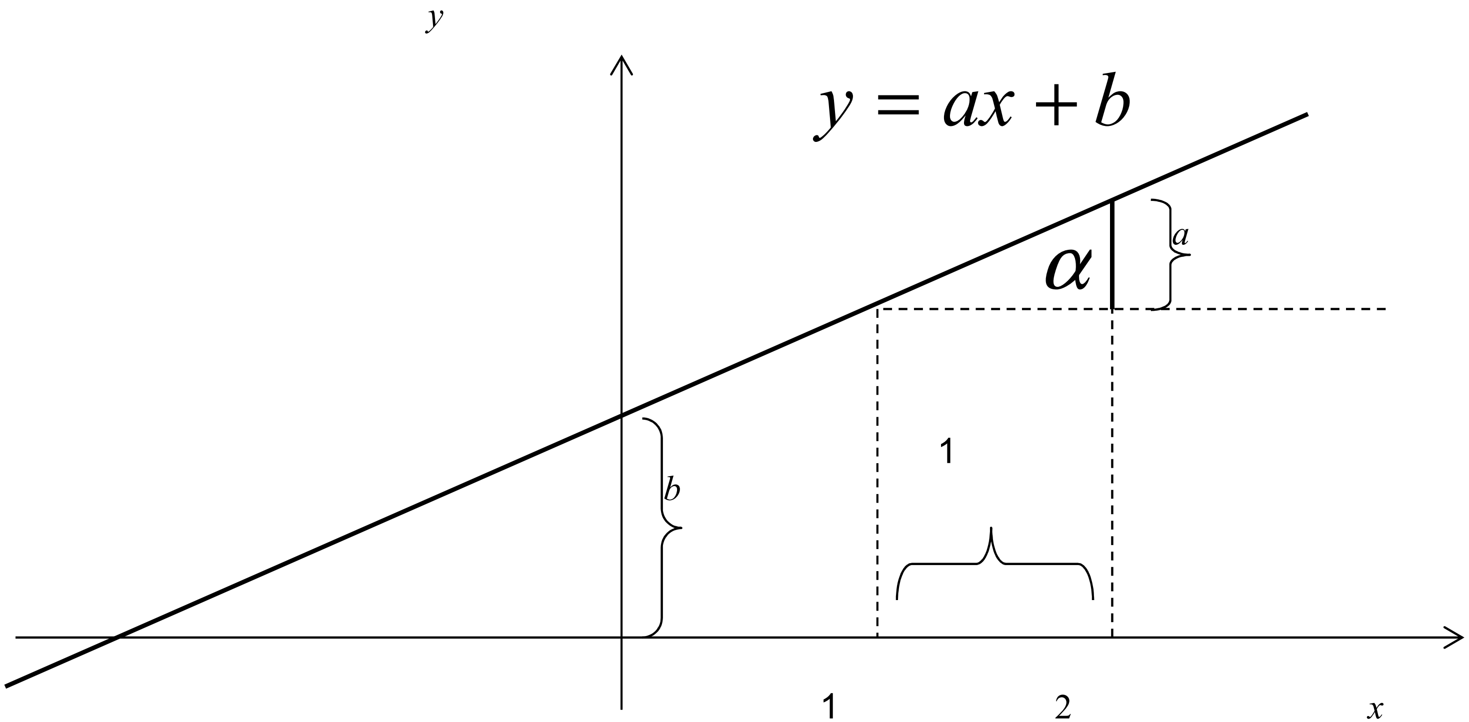
Lineární funkce je funkce ve tvaru:

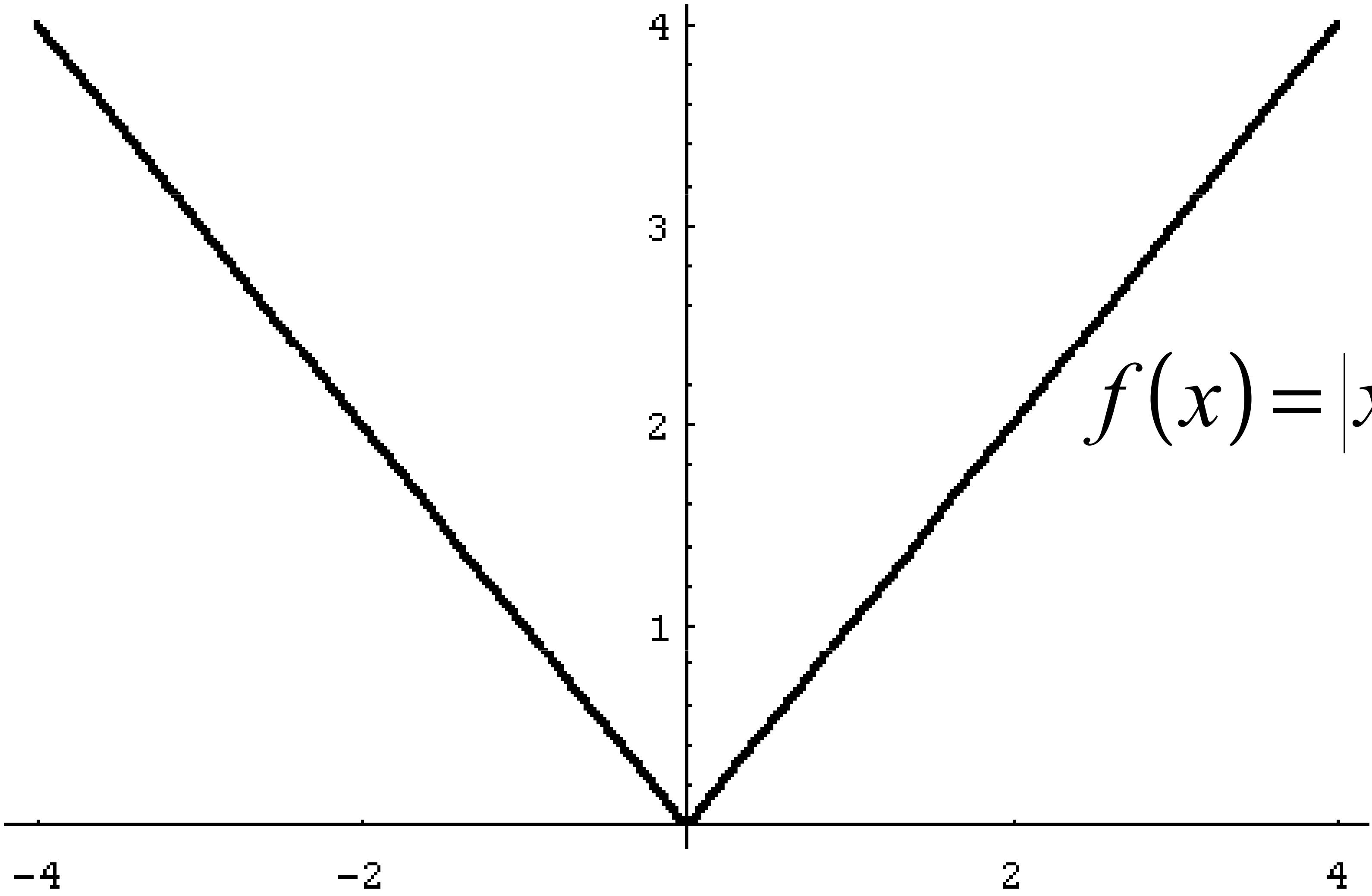
$$y = ax + b$$

Grafem této funkce je přímka. Jednotlivé koeficienty mají tento význam:

- ***a*** **-** směrnice přímky, která je grafem lineární funkce,
- *b* - úsek (vyťatý přímkou) na ose y

Graf lineární funkce .

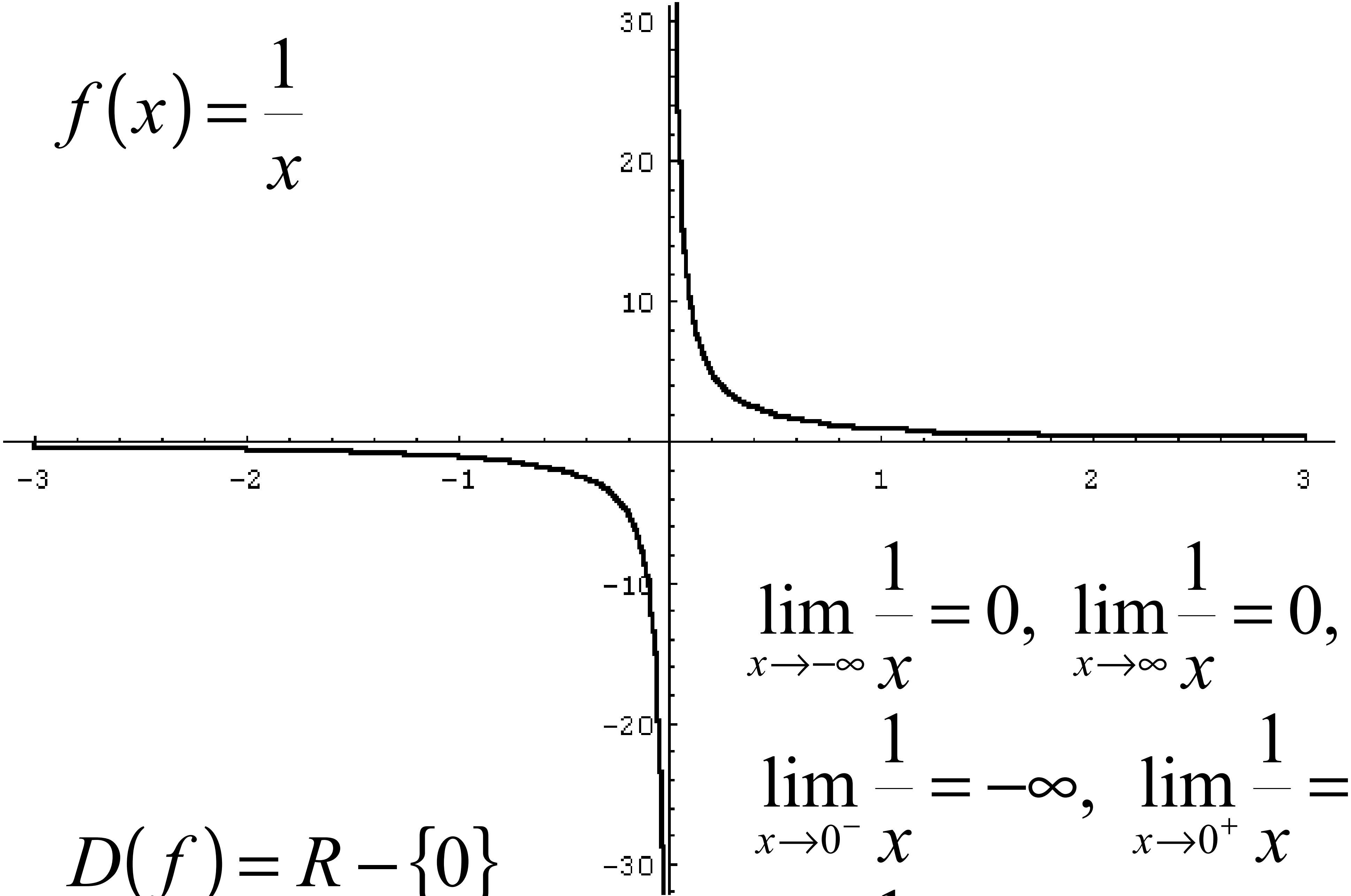




$$D(f)=R,\ \ H(f)=\langle 0,\infty),$$

$$\lim_{x\to-\infty}|x|=\infty,\ \ \lim_{x\to\infty}|x|=\infty$$

$$f(x)=\frac{1}{x}$$



$$\lim_{x\to-\infty}\frac{1}{x}=0,\quad \lim_{x\to\infty}\frac{1}{x}=0,$$

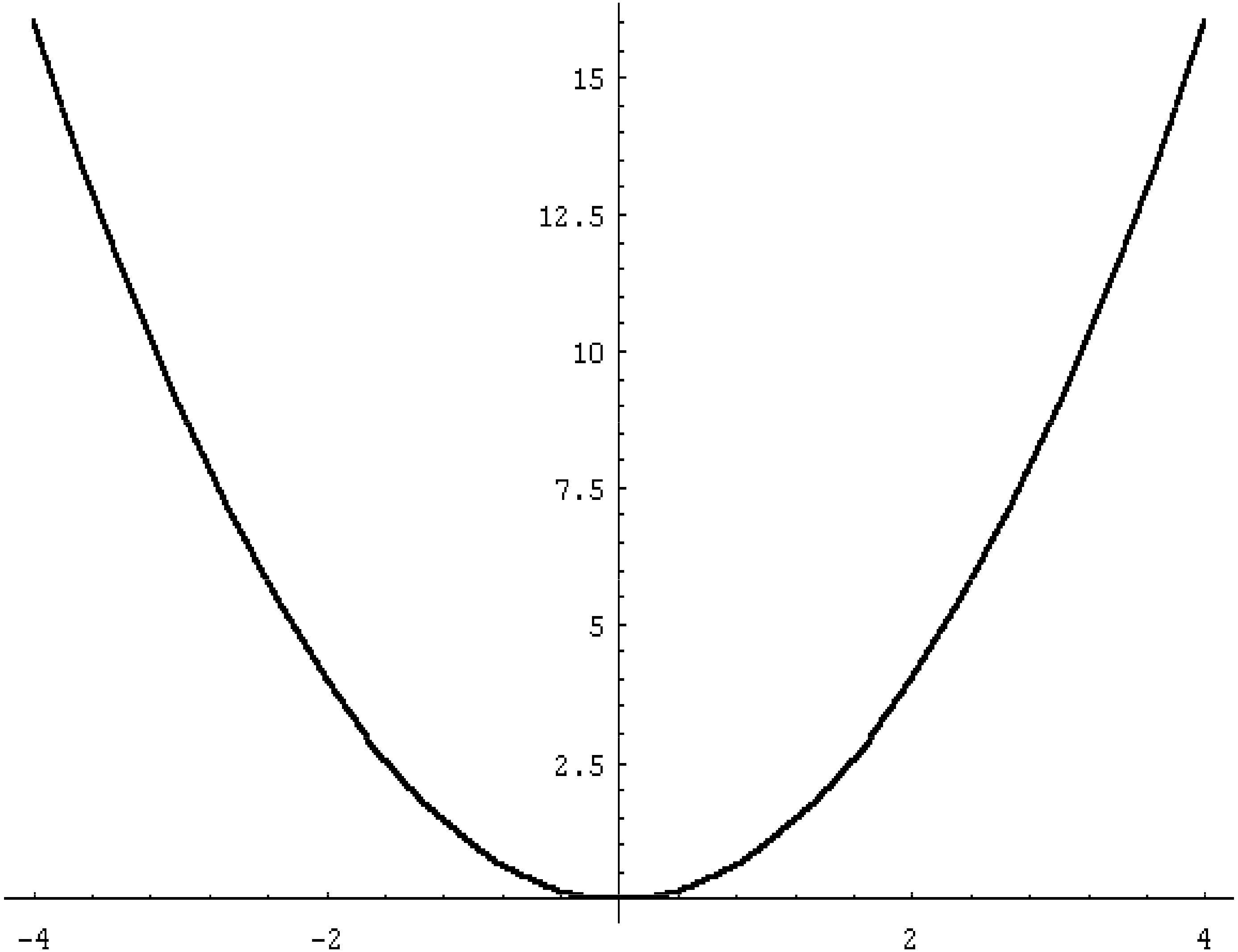
$$\lim_{x\to0^-}\frac{1}{x}=-\infty,\quad \lim_{x\to0^+}\frac{1}{x}=\infty,$$

$$D(f)=R-\{0\}$$

$$H(f)=R-\{0\}$$

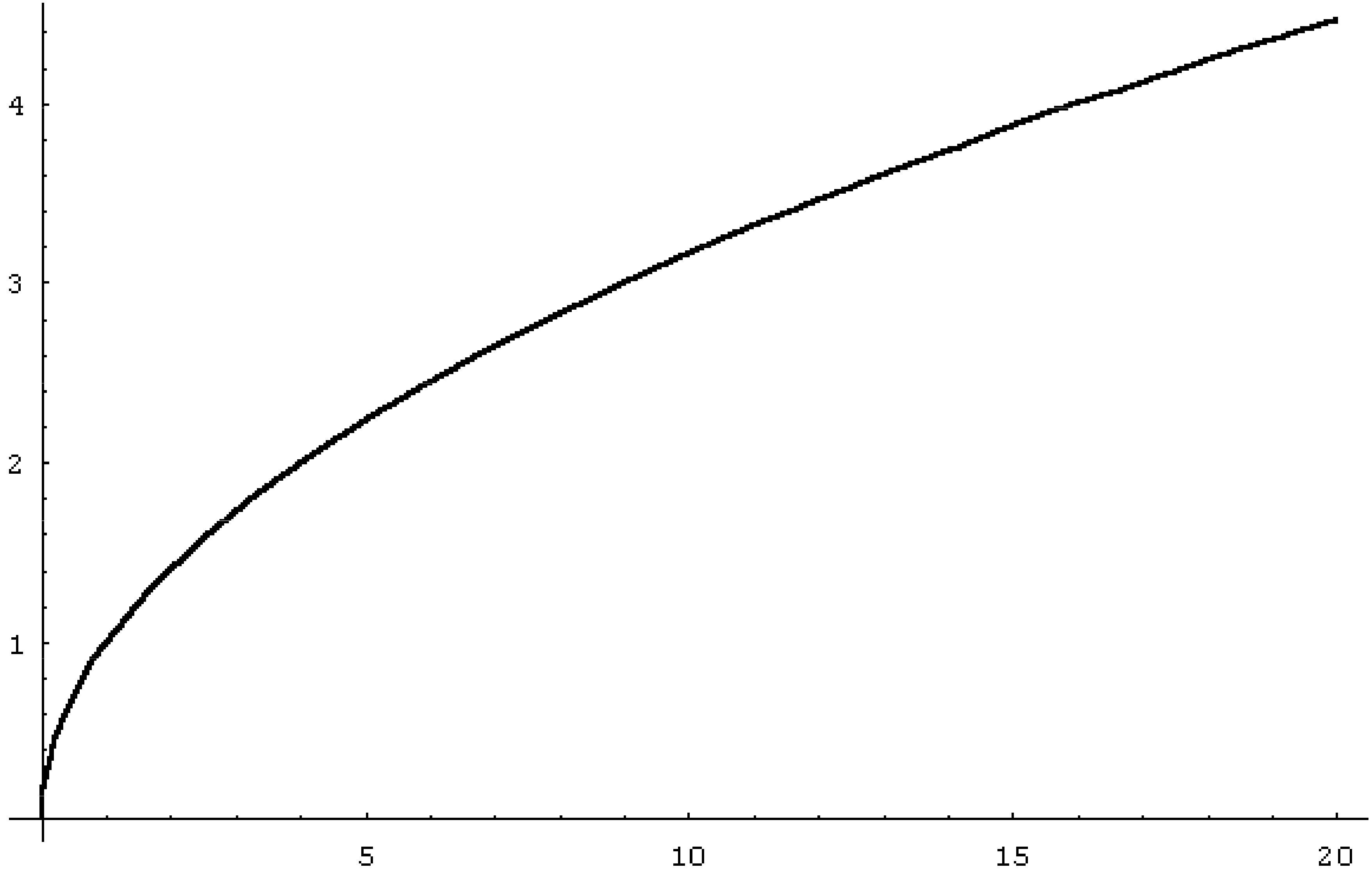
$\lim_{x\to0}\frac{1}{x}$ neexistuje

$$f(x) = x^2$$



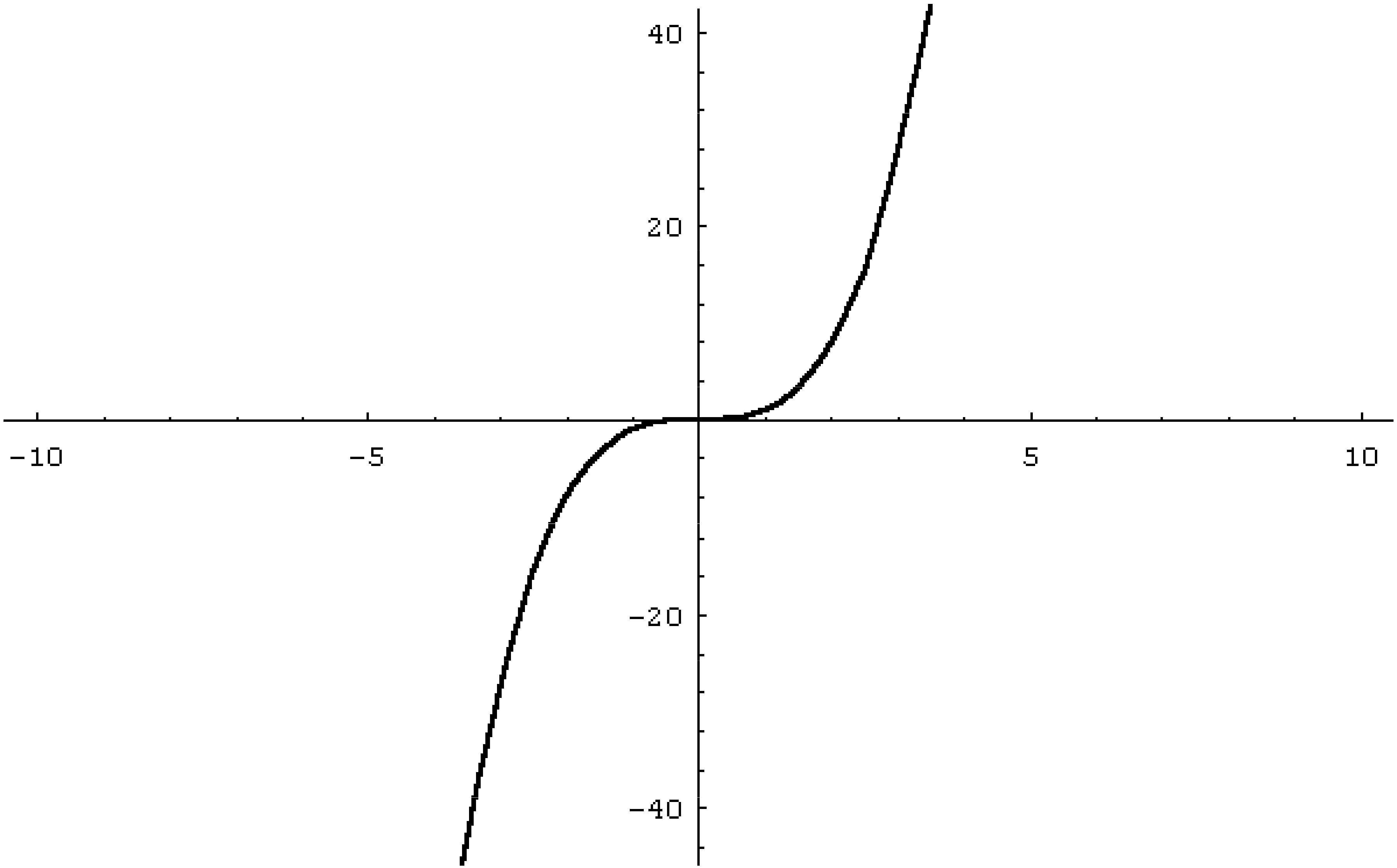
$$D(f) = R,\ H(f) = \langle 0, \infty),\ \lim_{x \to -\infty} x^2 = \infty,\ \lim_{x \to \infty} x^2 = \infty$$

$$f(x) = \sqrt{x}$$



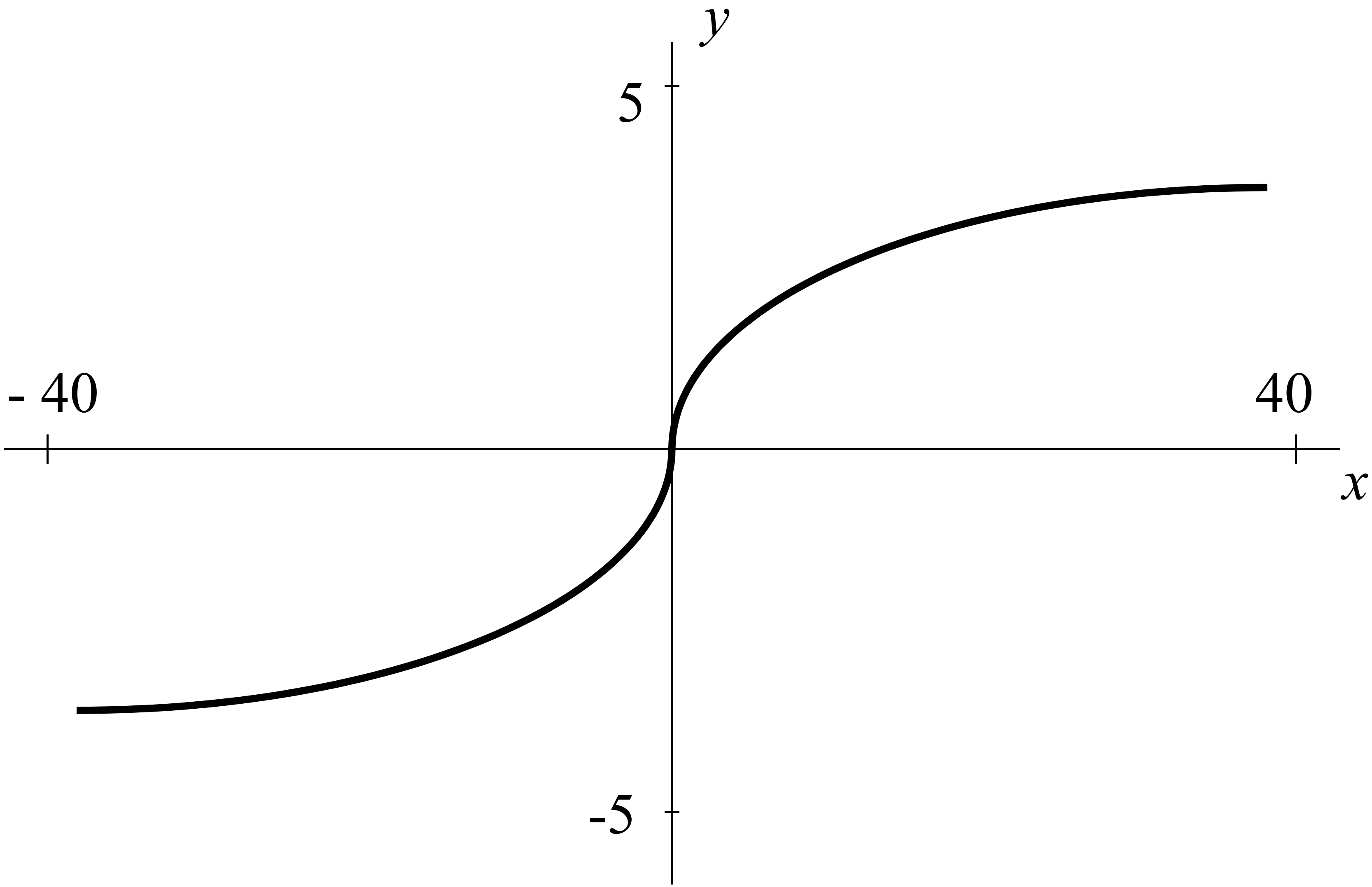
$$D(f) = \langle 0, \infty), \ H(f) = \langle 0, \infty), \ \lim_{x \to \infty} \sqrt{x} = \infty$$

$$f(x) = x^3$$



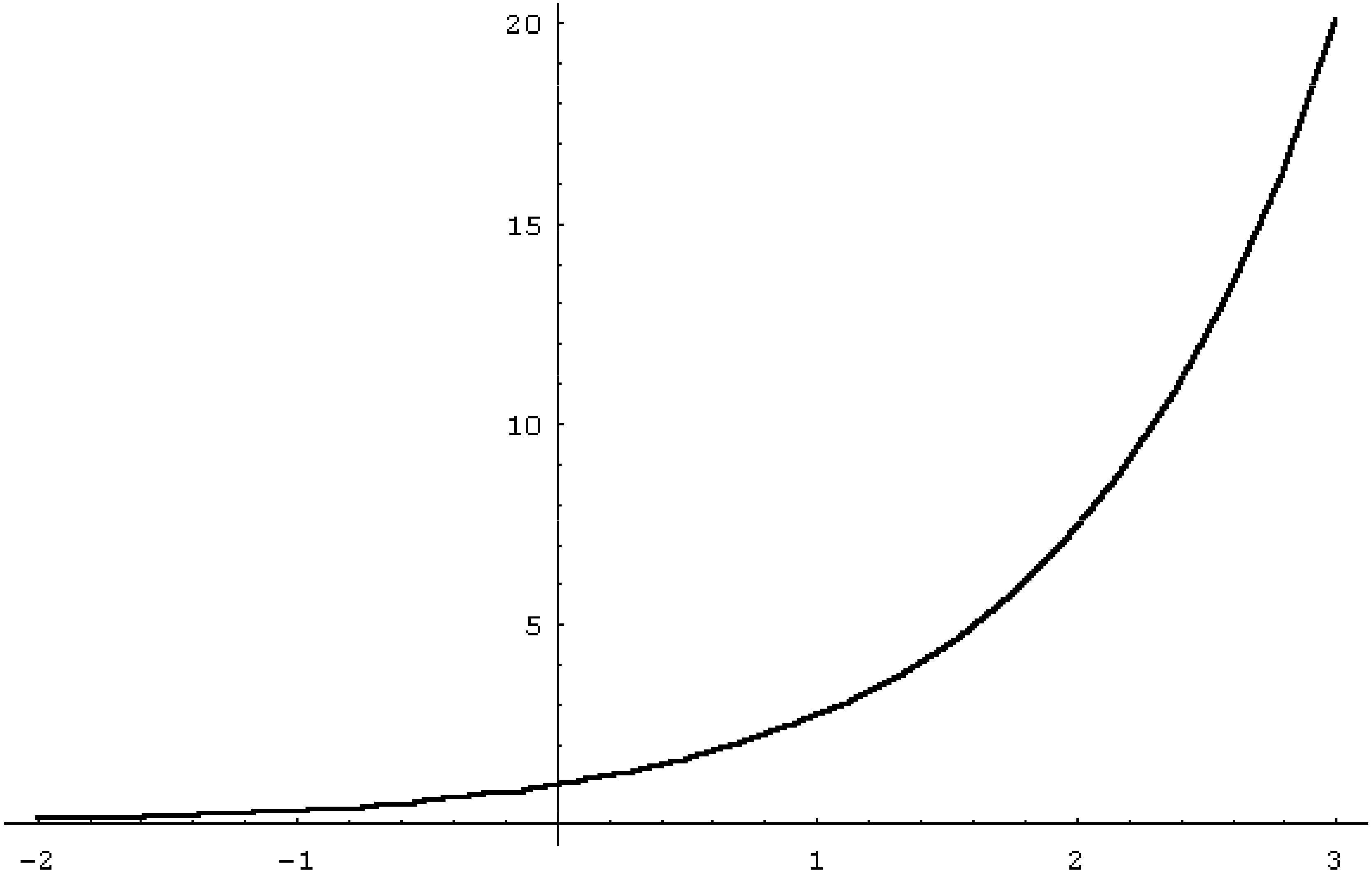
$$D(f) = R,\ \ H(f) = R,\ \ \lim_{x \to -\infty} x^3 = -\infty,\ \ \lim_{x \to \infty} x^3 = \infty$$

$$f(x)=\sqrt[3]{x}$$



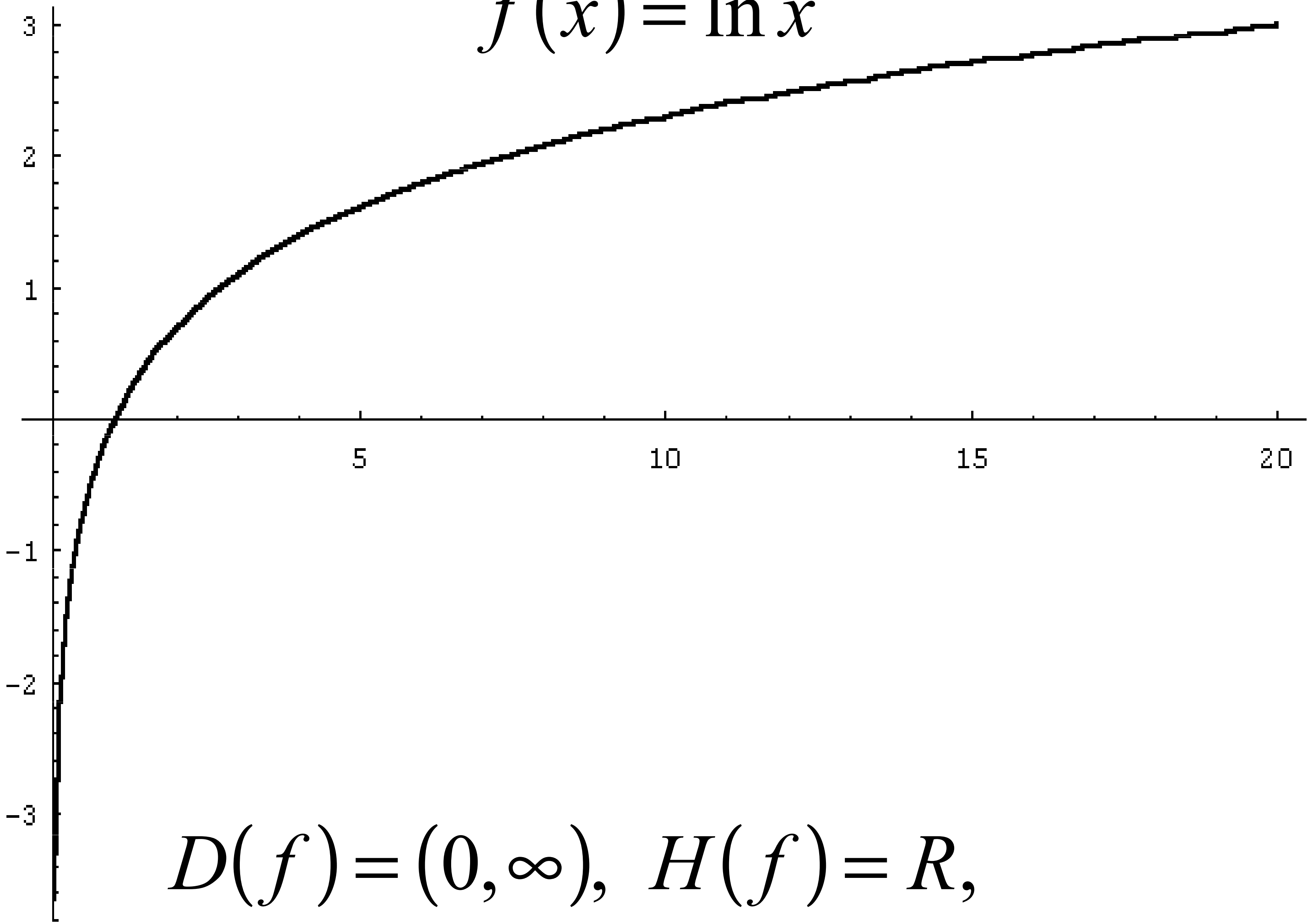
$$D(f)=R,\ \ H(f)=R,\ \ \lim_{x\to-\infty}\sqrt[3]{x}=-\infty,\ \ \lim_{x\to\infty}\sqrt[3]{x}=\infty$$

$$f(x)=e^x$$



$$D(f)=R,\ \ H(f)=(0,\infty),\ \ \lim_{x\to-\infty} e^x=0,\ \ \lim_{x\to\infty} e^x=\infty$$

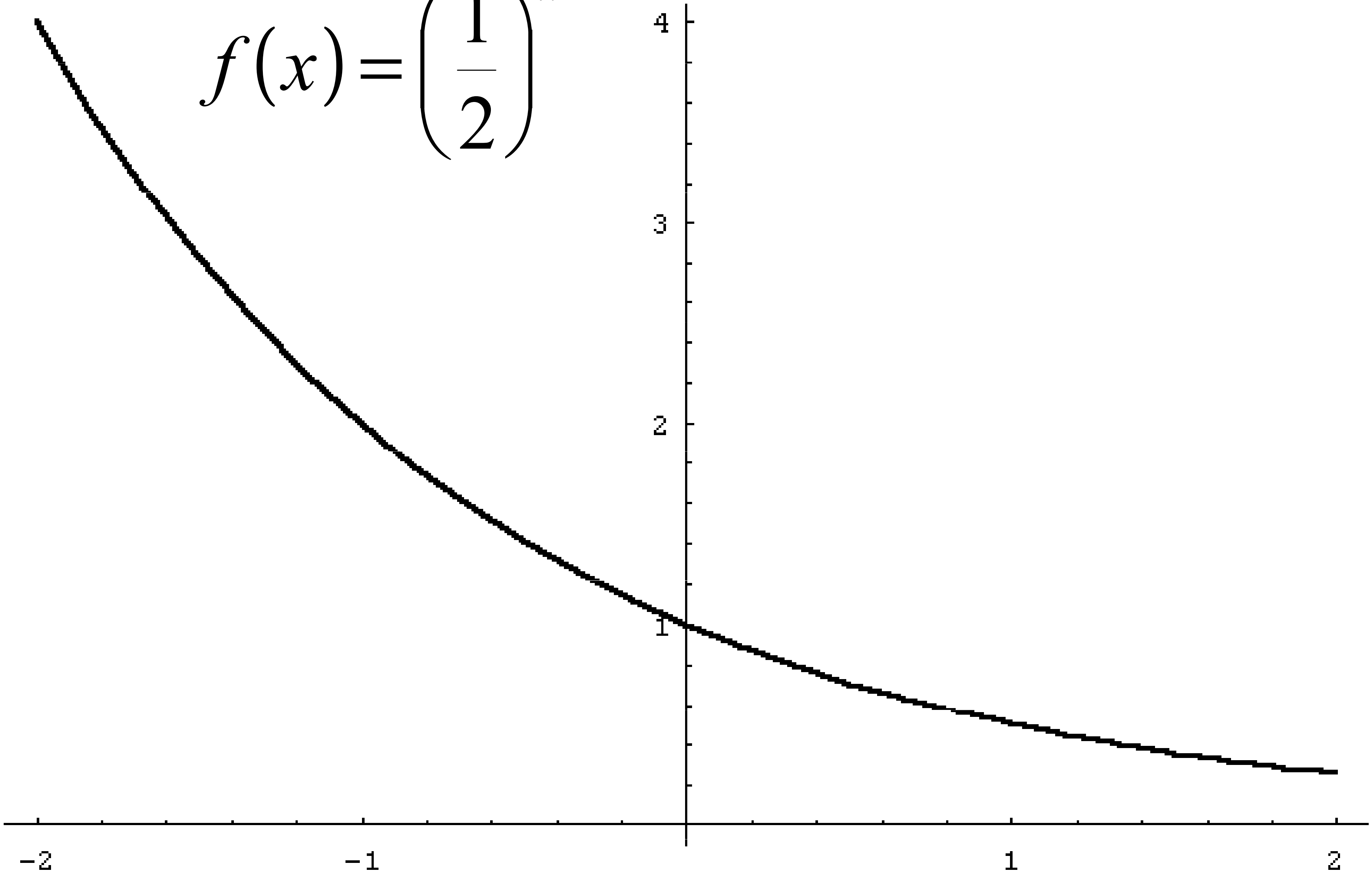
$$f(x) = \ln x$$



$$D(f) = (0, \infty), \;\; H(f) = R,$$

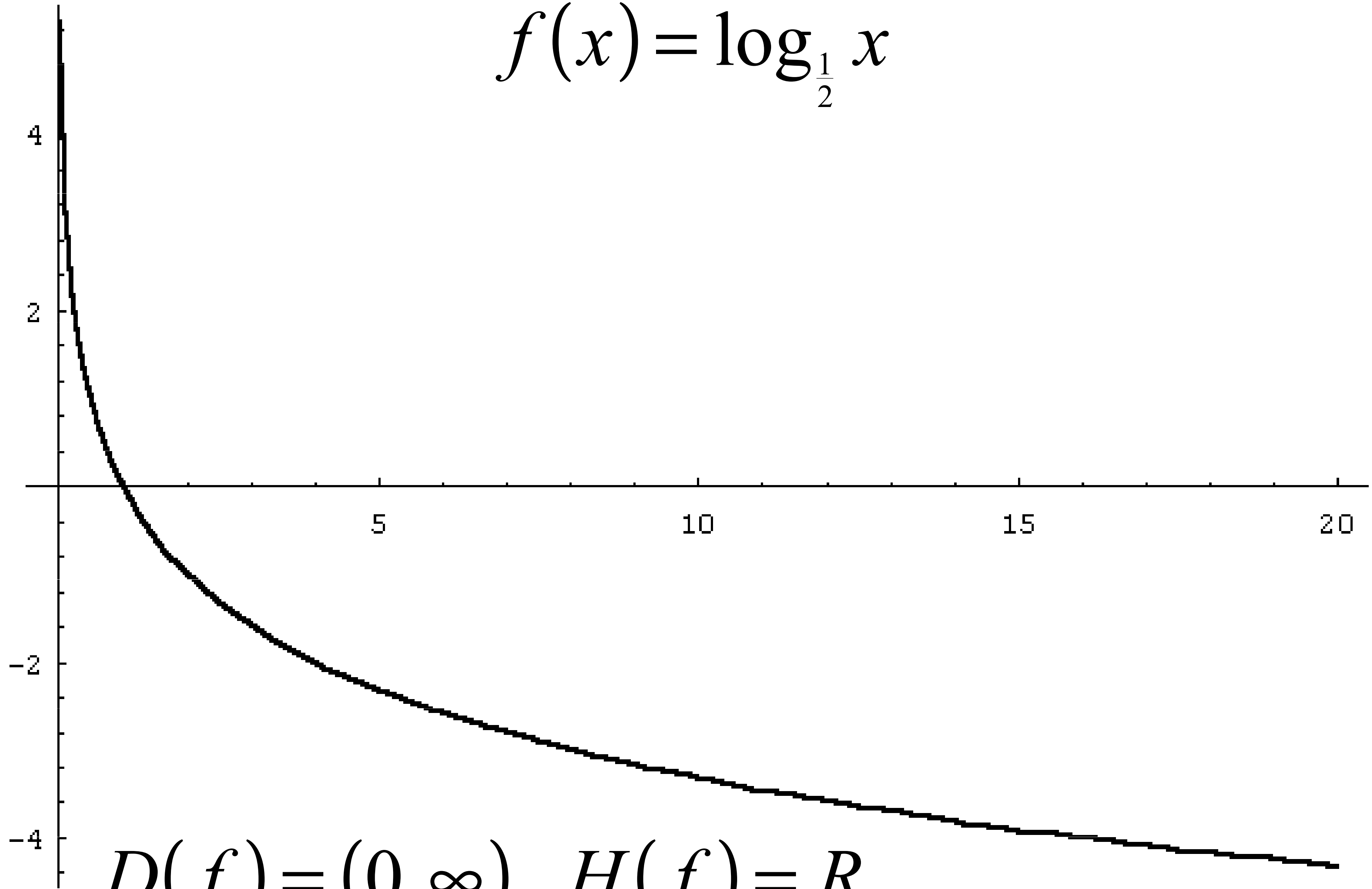
$$\lim_{x \to 0^+} \ln x = -\infty, \lim_{x \to \infty} \ln x = \infty$$

$$f(x)=\left(\frac{1}{2}\right)^x$$



$$D(f)=R,\ H(f)=(0,\infty),\ \lim_{x\to-\infty}\left(\frac{1}{2}\right)^x=\infty,\ \lim_{x\to\infty}\left(\frac{1}{2}\right)^x=0$$

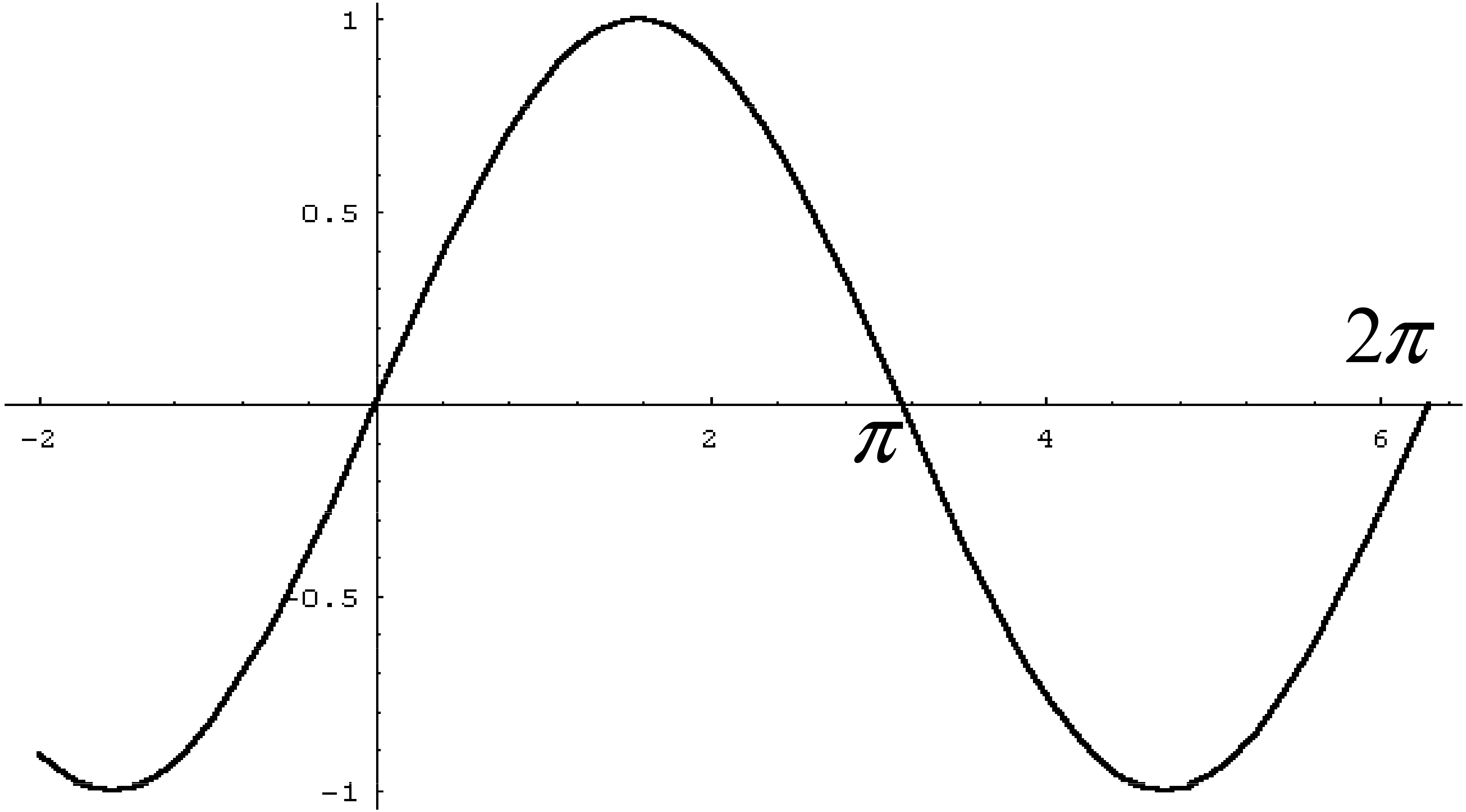
$$f(x)=\log_{\frac{1}{2}} x$$



$$D(f)=(0,\infty),\ H(f)=R,$$

$$\lim_{x\to 0^+}\log_{\frac{1}{2}} x=\infty,\ \lim_{x\to\infty}\log_{\frac{1}{2}} x=-\infty$$

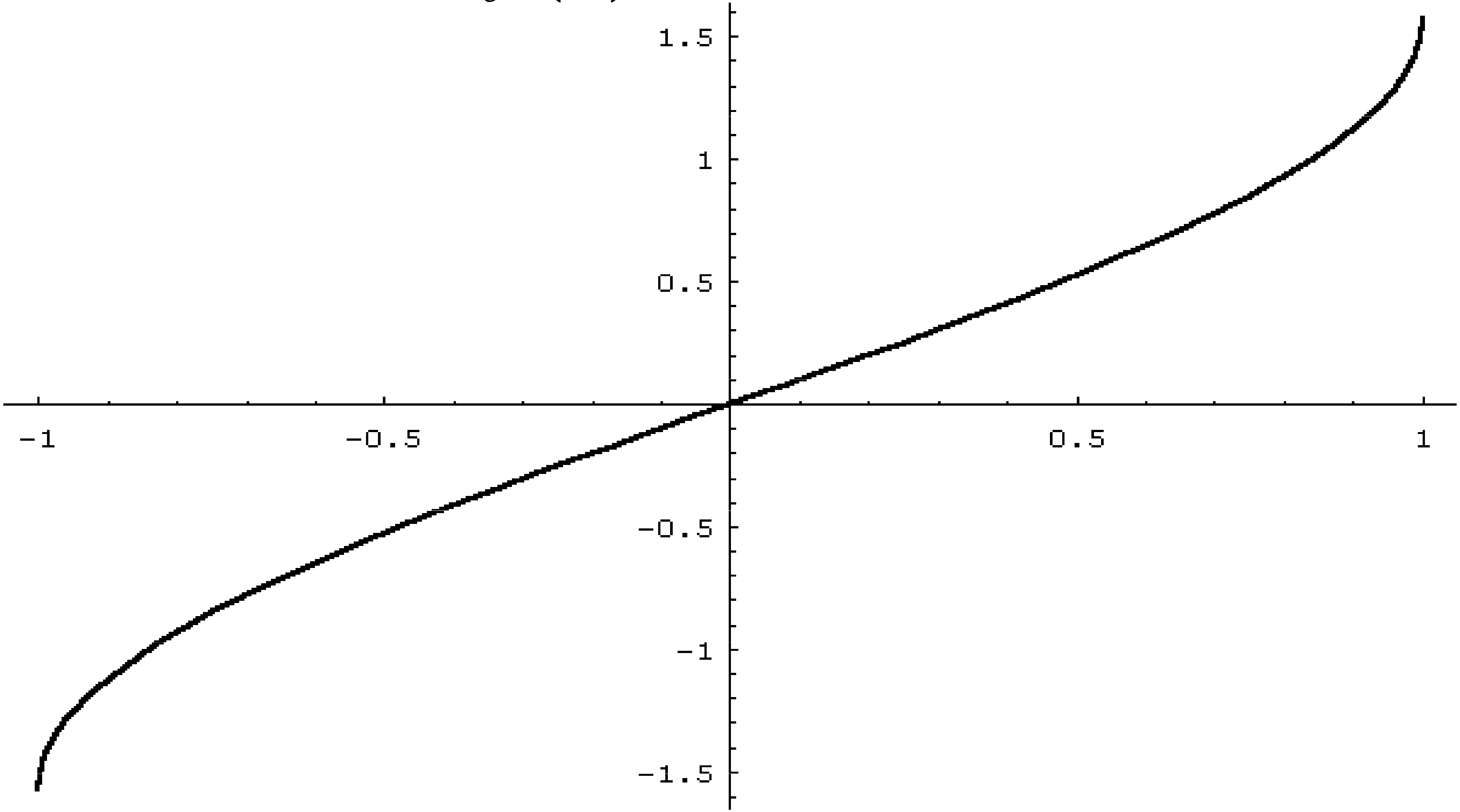
$$f(x) = \sin x$$



$$D(f) = R, \;\; H(f) = \langle -1,1 \rangle,$$

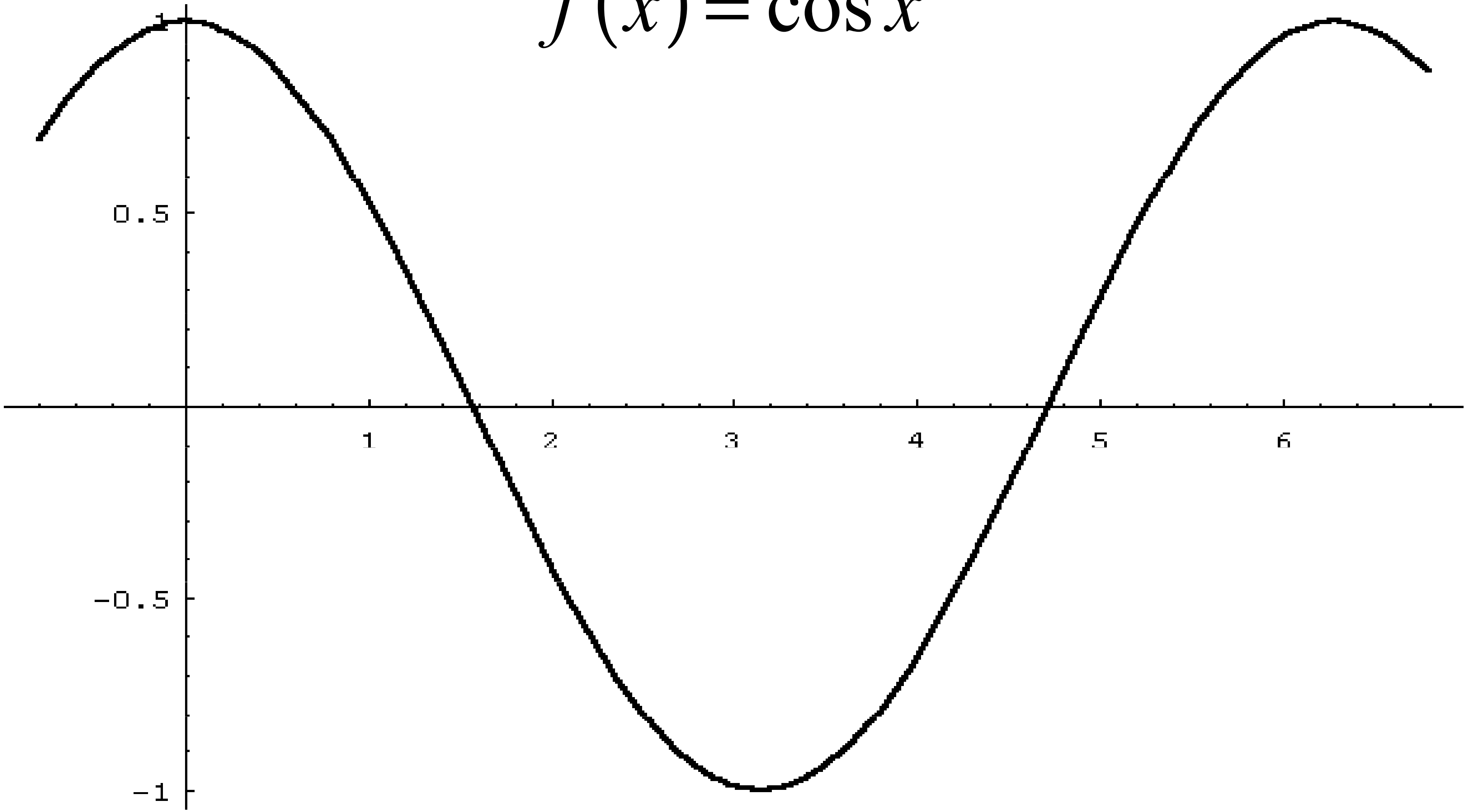
$$\lim_{x \to \infty} \sin x \;\; \text{neexist.} \qquad \lim_{x \to -\infty} \sin x \;\; \text{neexist.}$$

$$f(x) = \arcsin x$$



$$D(f) = \langle -1,1 \rangle,\ H(f) = \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$$

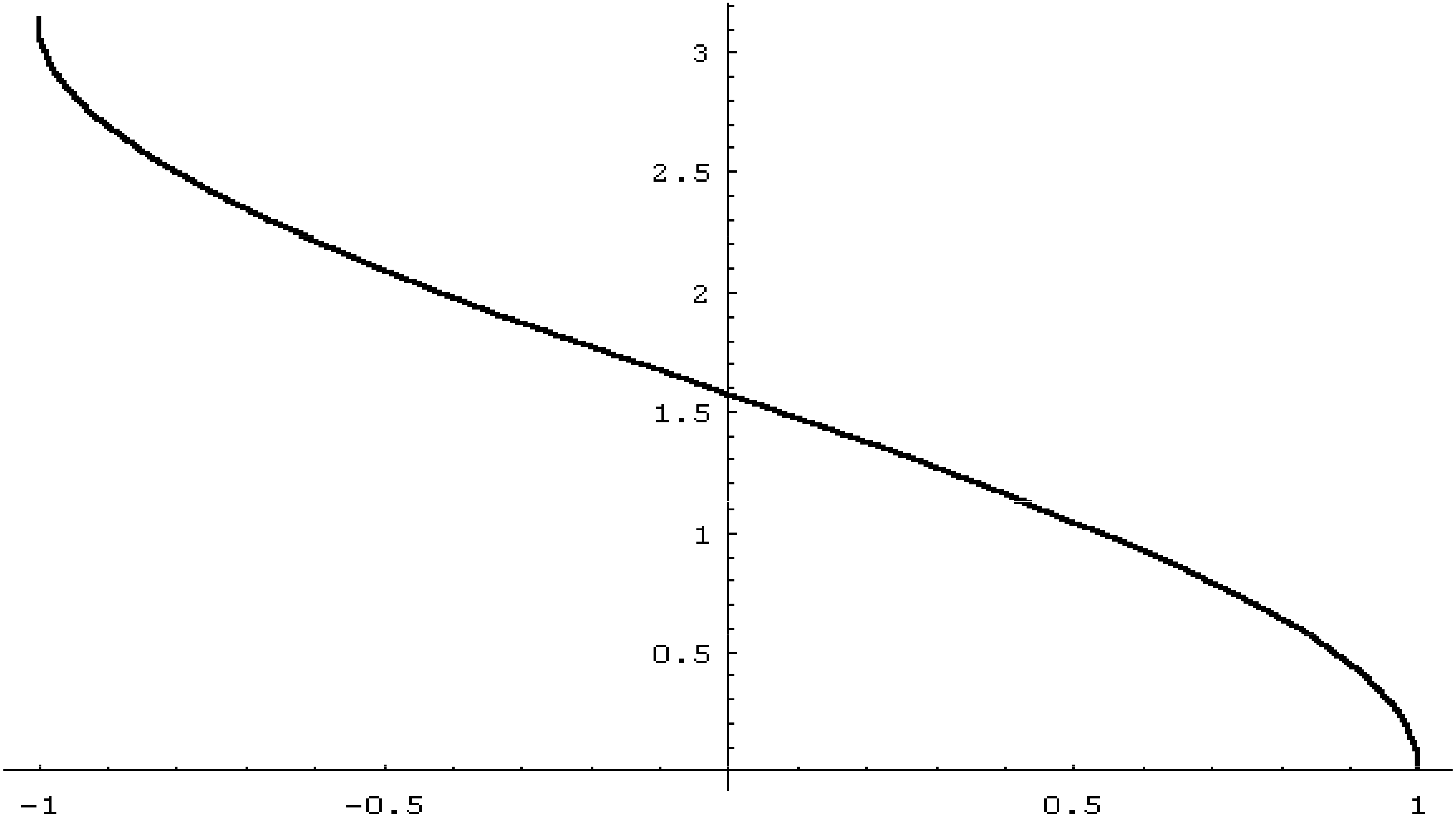
$$f(x) = \cos x$$



$D(f) = R, \; H(f) = \langle -1, 1 \rangle,$

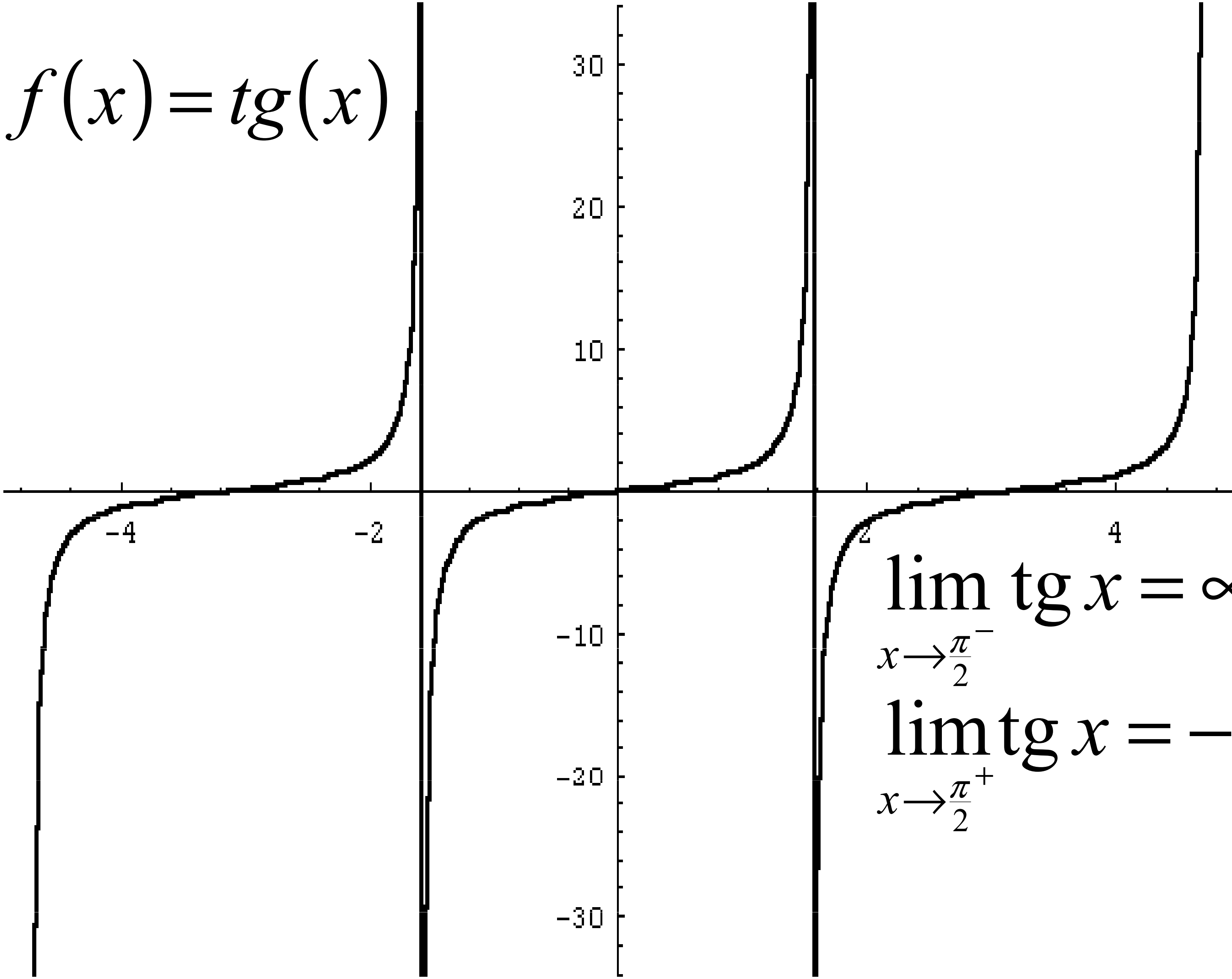
$\lim_{x \to \infty} \cos x$ neexistuje, $\lim_{x \to -\infty} \cos x$ neexistuje,

$$f(x) = \arccos x$$



$$D(f) = \langle -1, 1 \rangle, \;\; H(f) = \langle 0, \pi \rangle$$

$$f(x) = tg(x)$$

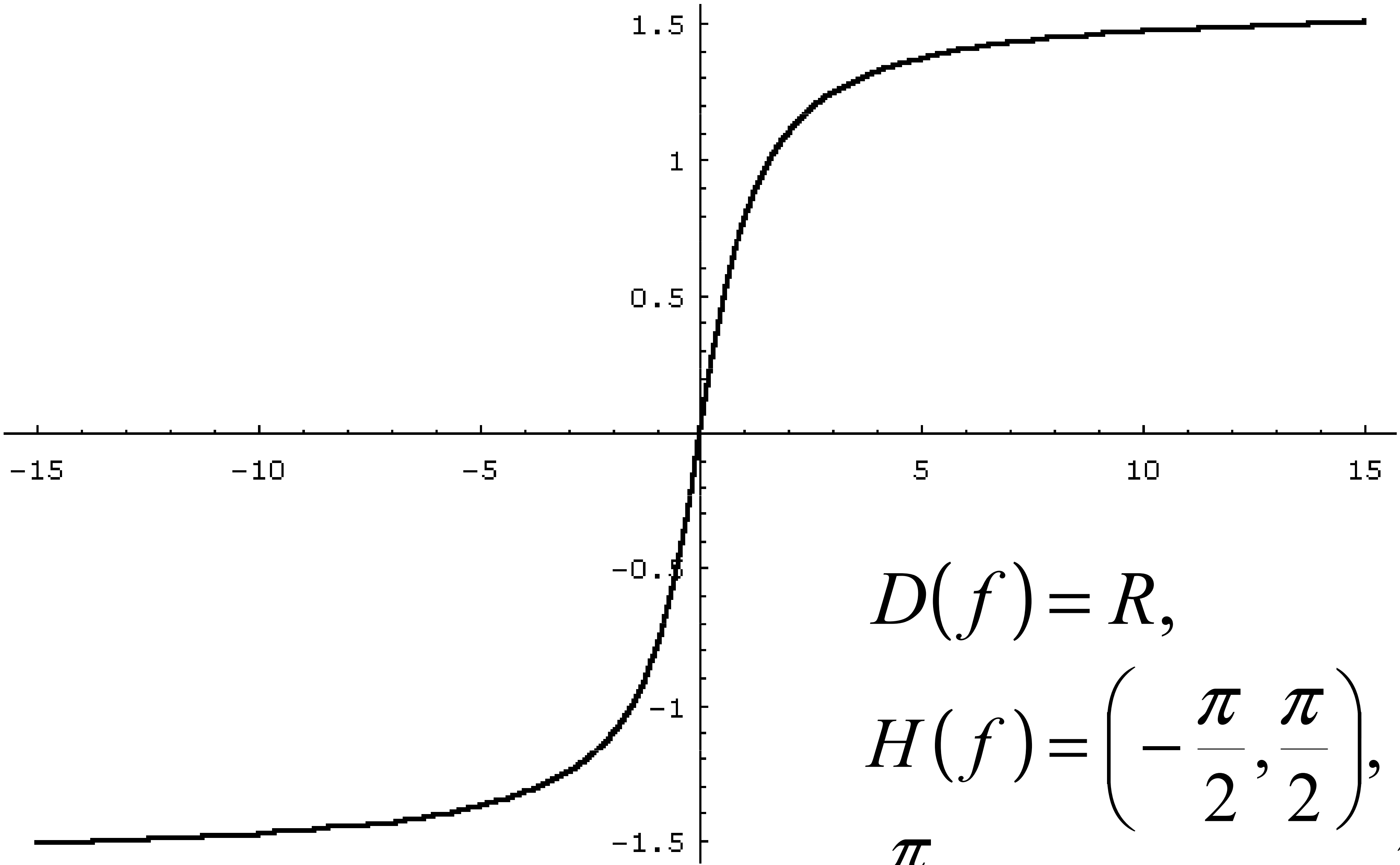


$$\lim_{x \to \frac{\pi}{2}^-} \operatorname{tg} x = \infty,$$

$$\lim_{x \to \frac{\pi}{2}^+} \operatorname{tg} x = -\infty,$$

$$D(f) = R - \left\{ (2k+1)\frac{\pi}{2}, k \in Z \right\}, H(f) = R$$

$$f(x) = \operatorname{arctg} x$$

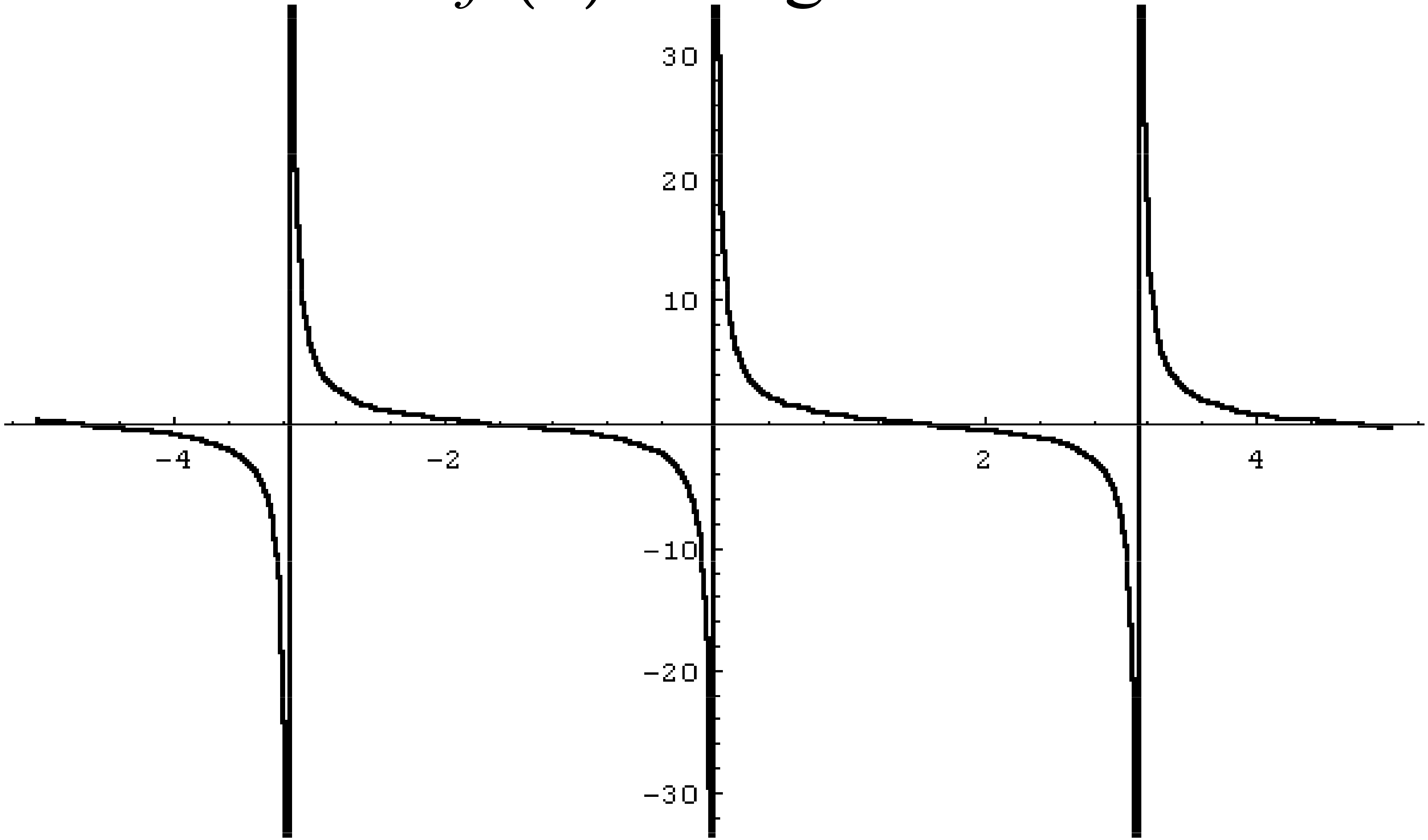


$$D(f) = R,$$

$$H(f) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right),$$

$$\lim_{x \to -\infty} \operatorname{arctg} x = -\frac{\pi}{2}, \ \lim_{x \to \infty} \operatorname{arctg} x = \frac{\pi}{2}$$

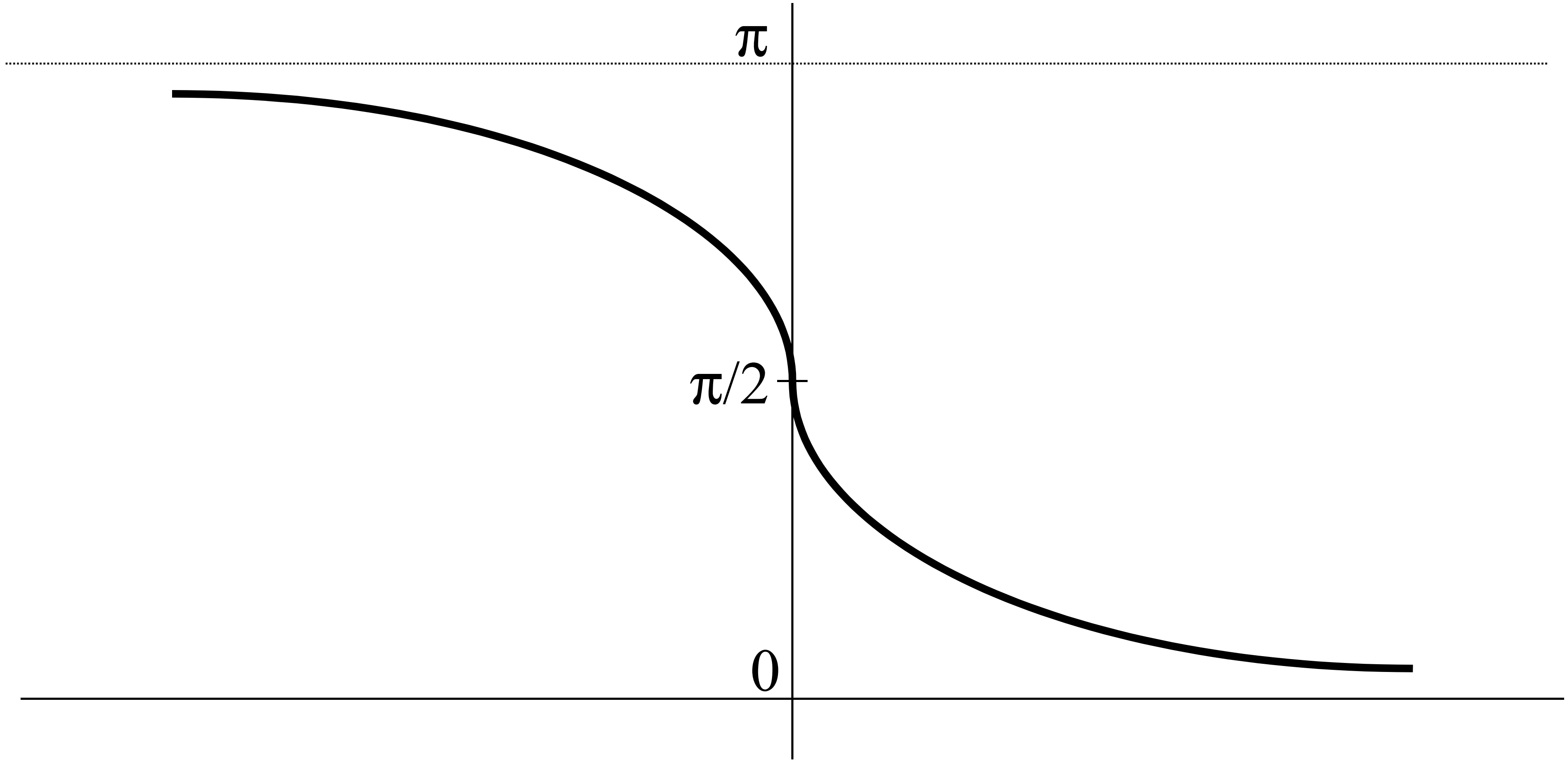
$$f(x)=\operatorname{cotg} x$$



$$D(f)=R-\{k\pi, k\in Z\},\ H(f)=R$$

$$\lim_{x\to 0^-}\operatorname{cotg}x=-\infty,\ \lim_{x\to 0^+}\operatorname{cotg}x=\infty,\ \lim_{x\to 0}\operatorname{cotg}x \text{ neexist.}$$

$$f(x) = \operatorname{arccotg} x$$



$$D(f) = R, \;\; H(f) = (0, \pi),$$

$$\lim_{x \to -\infty} \operatorname{arccotg} x = \pi, \;\; \lim_{x \to \infty} \operatorname{arccotg} x = 0$$

Příklad

Určete $D(f)$ funkce

$$y = \arcsin\left(2x-1\right)+\frac{3e^{1/x}}{\sqrt{x^2-3x+2}}$$