SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

Quantitative methods

Lecture 2

Mgr. Radmila Krkošková, Ph.D.
BAKVM

A real function of one real variable

- A **function** is a relation *f* between two sets X and Y such that each x from X is related to exactly one y from Y.
- We write *y = f(x).*
- **Examples**: $y = \quad +$

  $y =$
- The set **X** is called a **domain**, the set **Y** is called a **range** or a co-domain.
- In economics the domain usually consists of non-negative real numbers.

A graph of a function

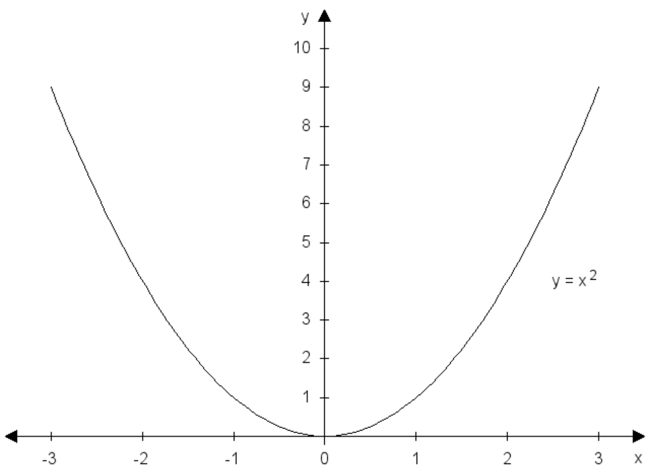
- A **graph** of a function is the collection of all ordered pairs (x,y) displayed in a two-dimensional plane.
- In a two dimensional plane it as a **curve**.

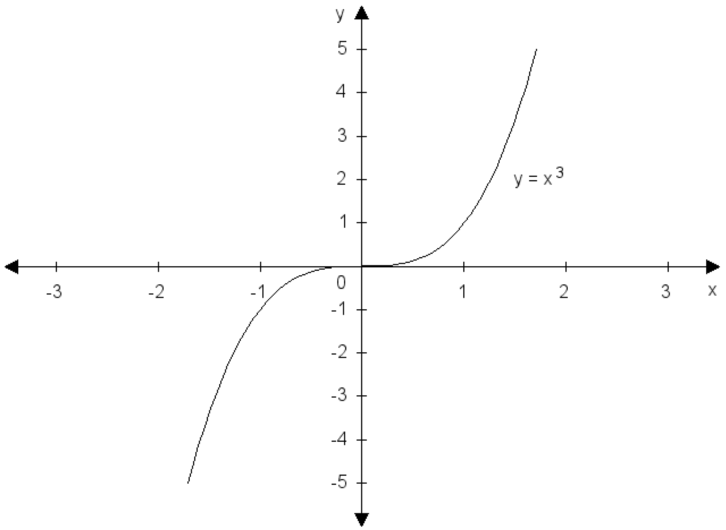
Examples:

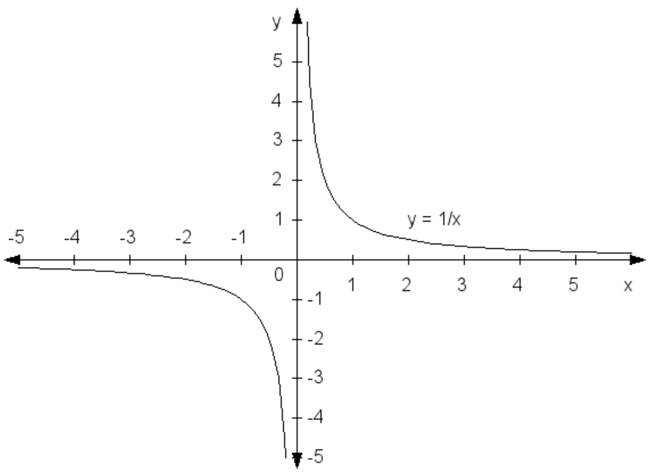
- Linear function: a line,
- Quadratic function: a parabola,
- Reciprocal function: a hyperbola.

Other functions are represented by more complex curves.

A graph of a function: examples







Elementary function properties

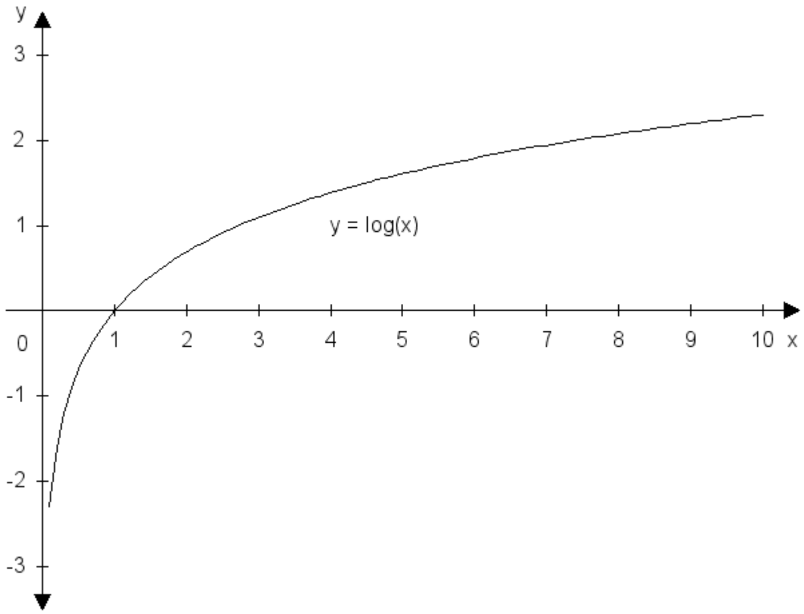
- Domain and range,
- Monotonicity (increasing, decreasing, non-increasing, non-decreasing, constant)
- Extremes (local or global maximum or minimum),
- Concavity and convexity,
- Inflection points,
- Bounded vs unbounded function,
- Even functions and odd functions,
- Peridiocity.

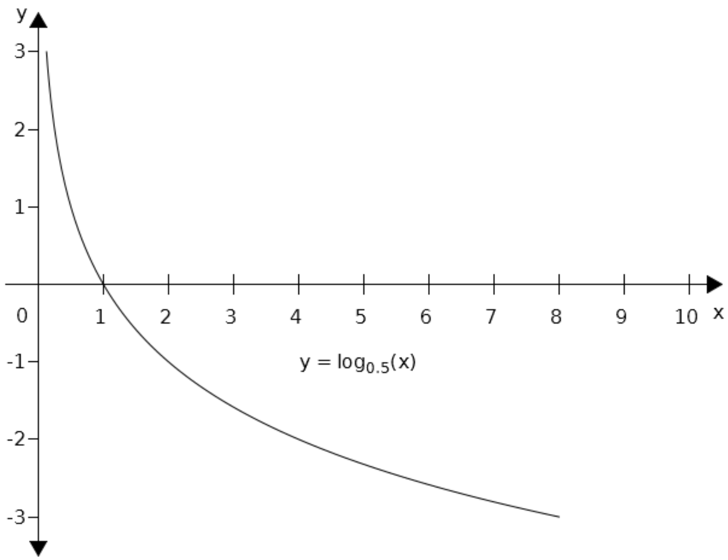
Elementary functions

- Linear function: $y = ax + b$,
- Quadratic function: $y = \quad + \quad +$
- Polynomial function: $y = \quad + \quad + \quad +$
- Linear reciprocal function: $y =$
- Logarithmic function: $y =$

In the logarithmic function, ***a*** is the so called a ***base***. The **decadic logarithm**: $a = 10$, the **natural logarithm**: $a = e = 2.718…$ For a larger than 1, the function is increasing, and for a smaller than 1 it is decreasing.

Graphs of logarithmic functions



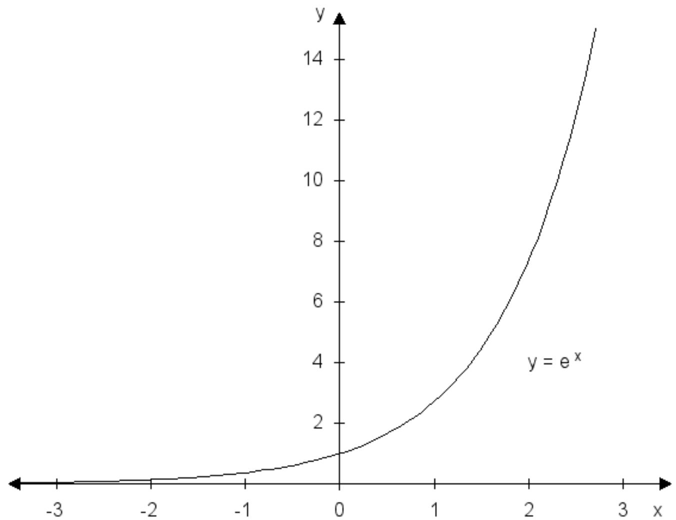


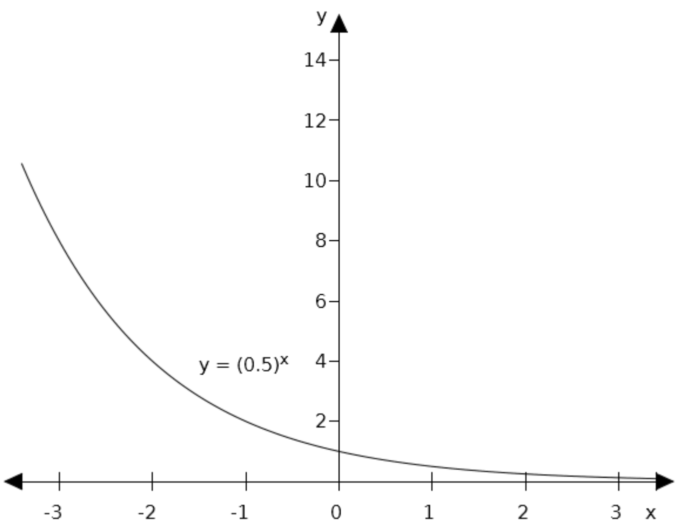
Elementary functions - continued

- Exponential function: $y =\quad >$
- Goniometric functions: y = sinx, y = cosx, y = tgx, y = cotgx.
- Cyclometric functions: y = arcsinx, y = arccosx, y = arctgx, y = arccotgx.

Notes: exponential and logarithmic functions are inverse to each other. The same applies to goniometric and cyclometric functions.

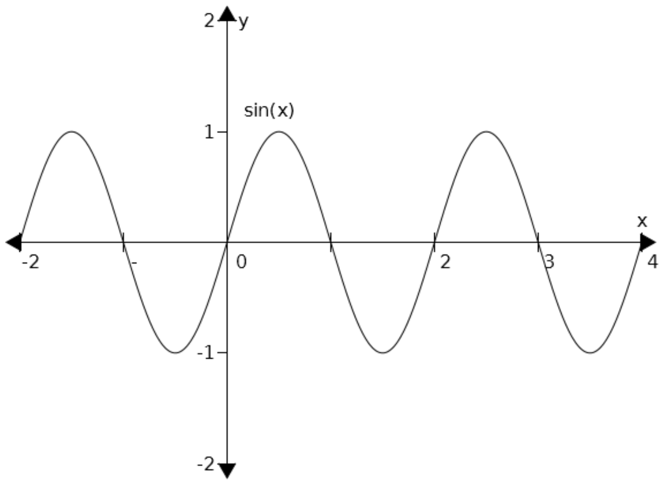
Graphs of exponential functions

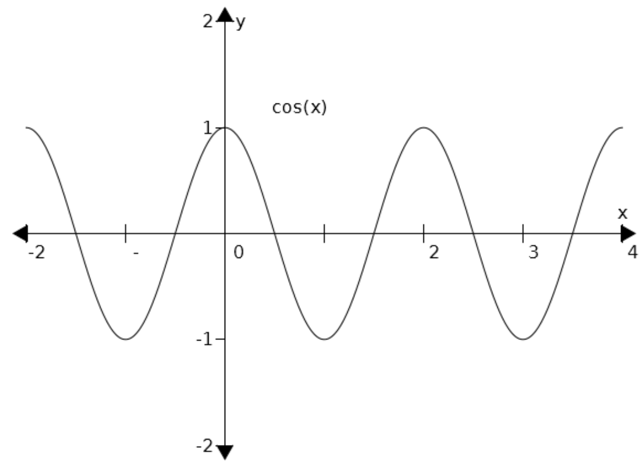


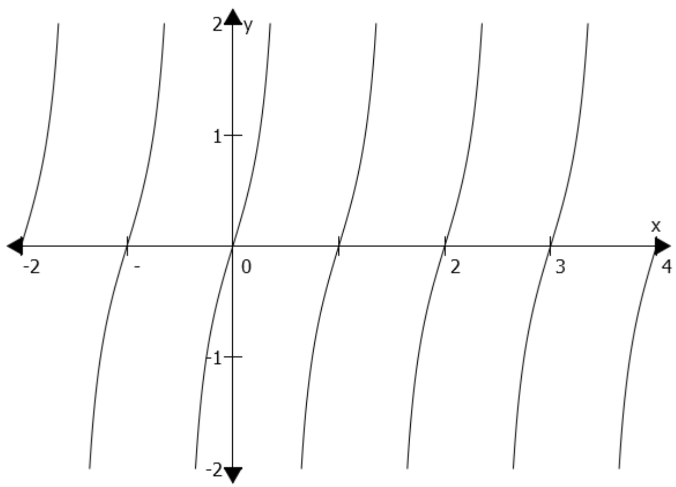


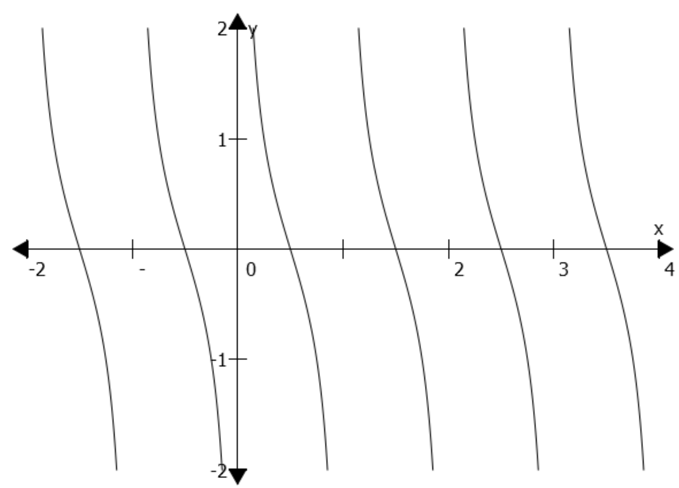
For $a > 1$, the exponential function is increasing, for $a < 1$ is decreasing.

Graphs of goniometric functions









Polynomials

In economics, various functions, such as demand or supply, are expressed by **polynomials**. Two main tasks when dealing with polynomials are transformation of a polyonomial into a product, and to find roots of a polynomial.

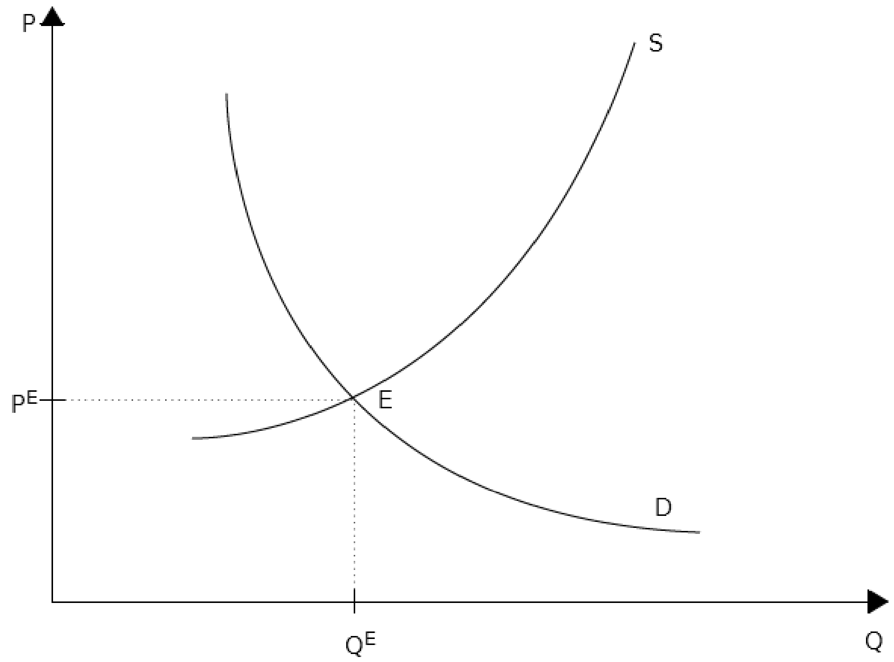
Let $P_n(x)$ denote a polynomial of a degree *n*.

Polynomial roots are such values of *x* that $P_n(x) =$.
The equation above can be solved via known formulas or identities such as $(a + \quad - \quad = \quad -$

Demand and supply function, equilibrium

- The demand function expresses relationship between a price of a good (P) and a demanded quantity (Q) by customers. Usually, the demand function is denoted as Q= D(P) or Q_D , and it is assumed this function is decreasing.
- The supply function expresses relationship between a price of a good (P) and a supplied quantity (Q) by sellers. Usually, the supply function is denoted as Q= S(P) or Q_S , and it is assumed this function is increasing.
- A point where demand is equal to supply, ant a market is cleared, is called an ***equilibrium***.

Demand and supply function, equilibrium – cont.



Solved problem 1

- Find the domain of the function f: $y = \quad - \quad$.

  Solution: the expression under the square root sign must be non-negative, therefore we obtain:

  $$2x - \quad \geq$$

  $$x \geq$$

- Hence, the domain is $D(f) = \quad \infty$.

Solved problem 2

- Find the domain of the function f: $y = \quad - \quad$.

  Solution: the expression in the logarithm must be positive, therefore we obtain:

  $$x^2 - \quad >$$

  We expand the term on the left hand side:

  $$x + \quad - \quad >$$

- From the last inequality it follows that -5 and 5 are the roots that divide the x line into three intervals. By checking the sign in each interval we obtain the final solution:

  $$D(f) = -\infty - \quad \cup \quad \infty$$

Solved problem 3

- Find the domain of the function f: $y =$ $-$.

  Solution: the expression in the arcsin is bounded by -1 from below and by 1 from above. Therefore, we obtain:

  $- < - <$

  By dividing this inequality into two simple linear inequalities we obtain:

  $2 <$ and $x <$

- Hence, we obtain the solution: $D(f) =$

Solved problem 4

Let us asssume that the demand and the supply functions are given as follows: $Q_D = \ -$, $Q_S = - +$.

Find the equilibrium.

Solution: in the equilibrium both functions are equal:

$$\mathrm{IU}\_ \quad -\_ \ \perp$$

Therefore, we obtain: P_E = 6, and Q_E = 4. Draw both function!

How will the situation change if there is a price floor P = 8?

Solved problem 5

Let us asssume that the demand and the supply functions are given as follows: $Q_D = \;-\;$, $Q_S = \;+\;$.

Find the equilibrium.

Solution: in the equilibrium both functions are equal:

$$24 - \quad - \quad +$$

$$18 -$$

Therefore, we obtain: $P_E = 3$, and $Q_E = 18$. Draw both function!

Problems to solve

1. Find the domain of the following functions:

$y = \quad -$

$y =$

$y = \quad -$

$y = \quad - \; - \; +$

$y = \quad +$

Problems to solve – cont.

2. Draw a graph of the following functions:

$$y = \quad +$$

$$y = - \quad -$$

$$y = \quad + \quad +$$

$$y = - \quad + \quad -$$

$$y = \quad +$$

$$y = \quad -$$

$$y = \frac{x}{+}$$

Problems to solve – cont.

3. For the given functions of demand and supply find the equilibrium both geometrically and algebraically:

$$D(P) = \quad -$$
$$S(P) = \quad +$$

Thank you for your attention!