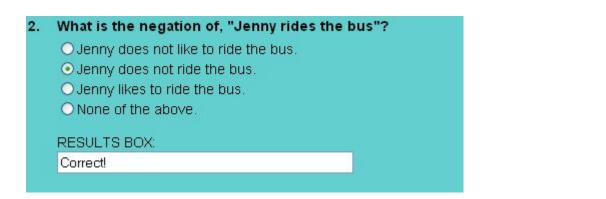
Solutions

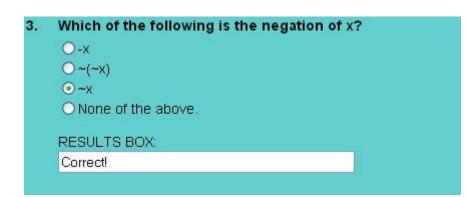
Lesson on Sentences and Negation

Which of the following is a closed sentence?
 Summer follows spring.
 A quarter is a coin.
 There are 360 days in a year.
 All of the above.
 RESULTS BOX:
 Correct!

Each of these sentences is a closed sentence (an objective statement which is either true or false).



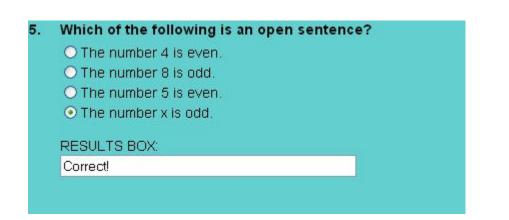
"Jenny does not ride the bus" is the negation of "Jenny rides the bus." The negation of p is "not p."



The statement $\sim x$ represents the negation of x.

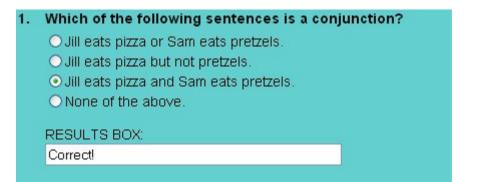
| Given: | a: A triangle is not a polygon. b: A square is a rectangle. | | | | | |
|-----------|--|--|--|--|--|--|
| | | | | | | |
| Problem: | Which of the following is the negation of "A triangle is not a polygon"? | | | | | |
| ⊙~(~b) | | | | | | |
| ⊙~a | | | | | | |
| ⊙a | Oa | | | | | |
| O None of | f the above. | | | | | |
| RESULTS | BOX: | | | | | |
| Correct | | | | | | |

The statement ~a represents the negation of a.

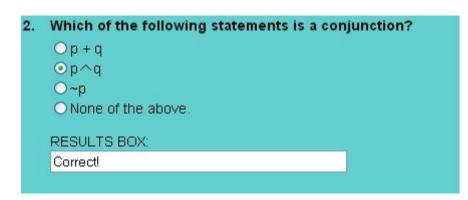


Choice 4 has a variable in it. An open sentence is a statement which contains a variable and becomes either true or false depending on the value that replaces the variable.

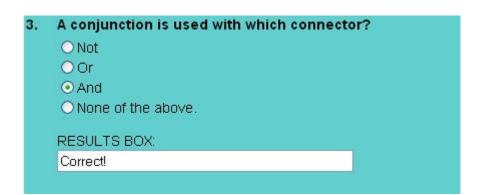
Lesson on Conjunction



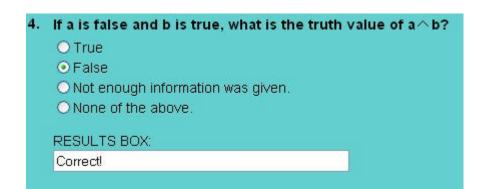
"Jill eats pizza and Sam eats pretzels" is a conjunction. A conjunction is a compound statement formed by joining two statements with the connector AND.



The conjunction "p and q" is symbolized by $p \land q$.



A conjunction is a compound statement formed by joining two statements with the connector AND.

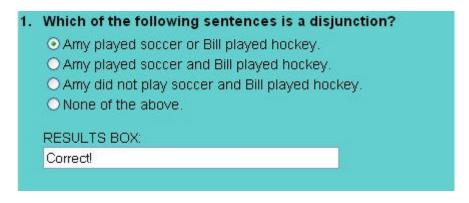


The truth value of $a \land b$ is false. A conjunction is true when both of its combined parts are true, otherwise it is false.

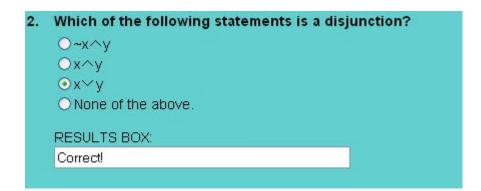
| Given: | r: y is prime. | | | | |
|-------------------------------------|---|--|--|--|--|
| | s: y is even. | | | | |
| Problem: | What is the truth value of r \land s when y is replaced by 2? | | | | |
| True | | | | | |
| OFalse | | | | | |
| ○ Not enough information was given. | | | | | |
| O None of the above. | | | | | |
| | | | | | |
| PESULTS | BEIX: | | | | |
| RESULTS Correct! | BOX: | | | | |

When y = 2, the statement r is true and the statement s is true (i.e., The number 2 is both prime and even). Therefore, the conjunction r \land s is true when y = 2.

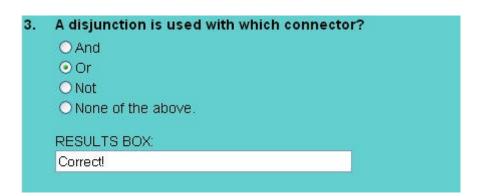
Lesson on Disjunction



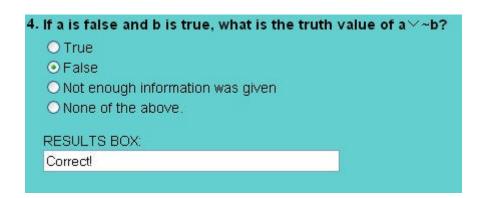
A disjunction is a compound statement formed by joining two statements with the connector OR.



The statement $x \lor y$ is a disjunction.



A disjunction is a compound statement formed by joining two statements with the connector OR.



If b is true then \sim b is false. A disjunction is false when both statements are false. Therefore, the disjunction a $\vee \sim$ b is false.

| 5. | Given: | r: y is prime. s: y is even. | | | | | |
|----|---------------------|--|--|--|--|--|--|
| | | | | | | | |
| | Problem: | Which of the following is a true statement when y is replaced by 3? | | | | | |
| | Or∨~s | | | | | | |
| | Or^~s | | | | | | |
| | Or∨s | | | | | | |
| | O All of the above. | | | | | | |
| | RESULTS | BOX: | | | | | |
| | Correct | | | | | | |

When y = 3, the statement r is true and the statement s is false. Therefore, all three choices list true statements.

| 1 | Which of the following is a conditional statement? |
|---|--|
| | • Amy plays soccer or Bill plays hockey. |
| | O Bill plays hockey when Amy plays soccer. |
| | If Amy plays soccer then Bill plays hockey. |
| | ○ None of the above. |
| | RESULTS BOX |
| | Correct |

A conditional statement is an if-then statement in which p is a hypothesis and q is a conclusion.

| 2. | Given: | r: You give me twenty dollars. s: I will be your best friend. | | | | |
|----|-----------|---|--|--|--|--|
| | | | | | | |
| | Problem: | Which of the following statements represents, "If you give me twenty dollars, then I will be your best friend"? | | | | |
| | Or∧s | | | | | |
| | ⊙r→s | | | | | |
| | ⊙s→r | | | | | |
| | O None of | the above. | | | | |
| | RESULTS | BOX: | | | | |
| | Correct! | | | | | |

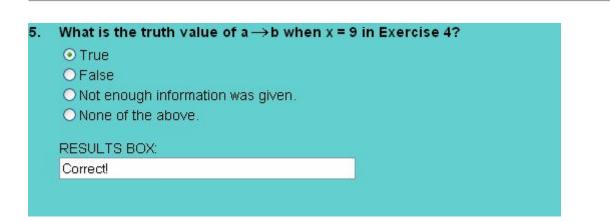
The hypothesis is r and the conclusion is s. The logical connector in a conditional statement is denoted by the symbol \rightarrow .

| 3. | What is the truth value of $r \! \rightarrow \! s$ when the hypothesis is false and the conclusion is true in Example 2? |
|----|--|
| | ⊙ True |
| | O False |
| | ONot enough information was given. |
| | O None of the above. |
| | RESULTS BOX: |
| | Correct |
| | |

The conditional is defined to be true unless a true hypothesis leads to a false conclusion.

| Given: | a: x is prime. b: x is odd. | | | | |
|-------------------------------------|---|--|--|--|--|
| | | | | | |
| Problem: | : What is the truth value of a $ ightarrow$ b when x = 2? | | | | |
| O True | | | | | |
| • False | | | | | |
| ○ Not enough information was given. | | | | | |
| ONone of the above. | | | | | |
| RESULTS | BOX: | | | | |
| Correct! | | | | | |

When x=2, hypothesis a is true and conclusion b is false. When a true hypothesis leads to a false conclusion, the conditional is false. Thus when x=2, conditional a \rightarrow b is false.

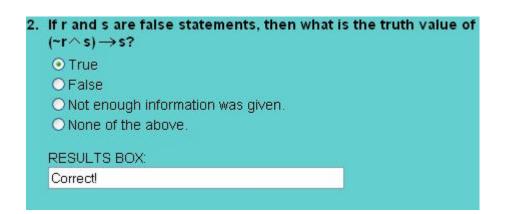


When x=9, hypothesis a is false and conclusion b is true. By definition, conditional $a \rightarrow b$ is true.

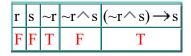
Lesson on Compound Statements

| Given: | a: 11 is prime. b: 11 is odd. | | |
|----------------------|---|--|--|
| | | | |
| Probler | n: Which of the following sentences represents (a $^{>}$ b) \rightarrow ~b? | | |
| ⊙lf 11 i ⊙lf 11 i | s prime and 11 is odd, then 11 is not odd. s prime or 11 is not odd, then 11 is not odd. s prime or 11 is odd, then 11 is not odd. of the above. | | |
| None | | | |
| RESULT | | | |

The compound statement $(a \lor b) \rightarrow \sim b$ is a conditional, where the hypothesis is the disjunction "a or b" and the conclusion is $\sim b$.



If r and s are false statements, then $(\sim r \land s) \rightarrow s$ is true as shown in the truth table below.



| What are the truth values of this statement? $({}^{\sim}x{}^{\checkmark}y){}^{\rightarrow}y$ |
|--|
| ⊙ {T, T, T, F} |
| ○ {T, T, T, T} |
| ○ {T, F, T, T} |
| O None of the above. |
| RESULTS BOX: |
| Correct |

The truth values of $(\neg x \lor y) \rightarrow y$ are shown in the truth table below.

| | | | $\sim x \lor y$ | $(\sim x \lor y) \rightarrow y$ |
|---|---|---|-----------------|---------------------------------|
| Т | Т | F | Т | Т |
| | F | | F | Т |
| F | Т | Т | Т | Т |
| F | F | Т | Т | F |

| What are the truth values of this statement? ${\sim}p{\rightarrow}(p{\wedge}{\sim}q$ |
|--|
| O {T, F, T, F} |
| O {F, T, F, T} |
| ⊙ {T, T, F, F} |
| O None of the above. |
| RESULTS BOX: |
| Correct |

The truth values of $\sim p \rightarrow (p \wedge \sim q)$ are {T, T, F, F},

as shown in the truth table below.

| p | q | ~p | ~q | p^~q | $\sim p \rightarrow (p \land \sim q)$ |
|---|---|----|----|------|---------------------------------------|
| Т | Т | F | F | F | Т |
| Т | F | F | Т | Т | Т |
| F | Т | Т | F | F | F |
| F | F | Т | Т | F | F |

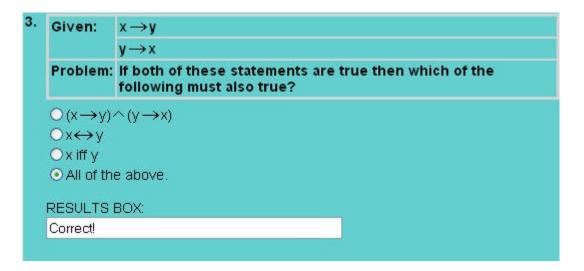
Lesson on Biconditional Statements

| Given: | a: y - 6 = 9 | | | | | | |
|--|----------------------|--|--|--|--|--|--|
| | b: y = 15 | | | | | | |
| Problem: The biconditional a↔b represents which of the follow sentences? | | | | | | | |
| O If y - 6 = 9, then y = 15. | | | | | | | |
| • y - 6 = 9 if and only if y = 15. | | | | | | | |
| O If y = 15, then y - 6 = 9. | | | | | | | |
| | O None of the above. | | | | | | |
| and the second | the above. | | | | | | |
| and the second | | | | | | | |

Biconditional $p \leftrightarrow q$ represents "p if and only if q," where p is a hypothesis and q is a conclusion.

| | Given: | r: 11 is prime. | | | | |
|---|----------------------------------|---|--|--|--|--|
| | | s: 11 is odd. | | | | |
| | Problem: | The biconditional $r \!$ | | | | |
| | Olf 11 is p | prime, then 11 is odd. | | | | |
| | Olf 11 is odd, then 11 is prime. | | | | | |
| | ● 11 is prime iff 11 is odd. | | | | | |
| | O None of the above. | | | | | |
| F | RESULTS | BOX: | | | | |
| | | | | | | |

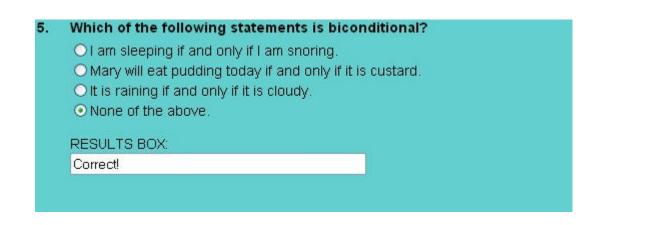
The hypothesis is "11 is prime" and the conclusion is "11 is odd". So $r \leftrightarrow s$ represents, "11 is prime if and only 11 is odd." The "if and only if" is abbreviated with "iff" in choice 3.



When proving the statement p iff q, it is equivalent to proving both of the statements "if p, then q" and "if q, then p". Since these conditionals were given in the problem, $x \leftrightarrow y$ is biconditional. Therefore, each statement listed in choice 1, 2 and 3 is true.

| Given: | m⇔n is biconditional |
|----------|--|
| Proble | n: Which of the following is a true statement? |
| ⊙ m is t | he hypothesis |
| ⊖ m is t | he conclusion |
| On is a | conditional statement |
| ⊖n is a | biconditional statement |
| RESULT | S BOX: |
| Correct! | |

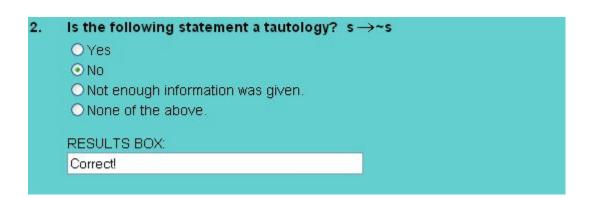
The biconditional $p \leftrightarrow q$ represents "p if and only if q", where p is a hypothesis and q is a conclusion. So m is the hypothesis of $m \leftrightarrow n$.



None of these statements is biconditional: one can sleep without snoring; Mary can eat pudding today that is not custard; it can be cloudy without any rain.

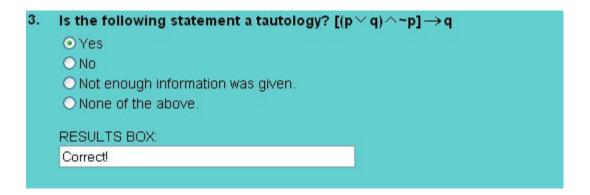
| What is the truth value of r $arsigma$ ~r? | |
|--|--|
| ⊙ True | |
| ○ False | |
| O Not enough information was given. | |
| ○None of the above. | |
| RESULTS BOX: | |
| Correct | |

A compound statement, that is always true regardless of the truth value of the individual statements, is defined to be a tautology. The disjunction of a statement and its negation is a tautology.



No, the conditional statement $s \rightarrow \sim s$ is not a tautology. See the truth table below.

| s | ~s | $s \rightarrow \sim s$ |
|---|----|------------------------|
| Т | F | F |
| F | Т | Т |



Yes, the statement $[(p \lor q) \land \neg p] \rightarrow q$

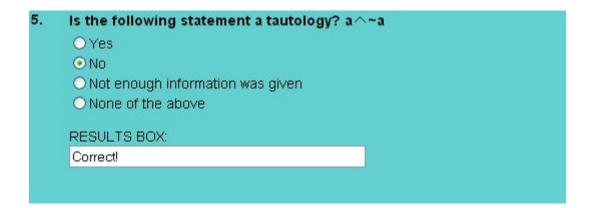
is a tautology since its truth values are $\{T, T, T, T\}$ as shown in the truth table below.

| | | | p∨q | (p∨q)^~p | $[(p \lor q) \land \neg p] \rightarrow q$ |
|---|---|---|-----|----------|---|
| Т | Т | F | Т | F | Т |
| | | F | Т | F | Т |
| F | Т | Т | Т | Т | Т |
| F | F | Т | F | F | Т |

| • Yes | |
|--------------|---------------------------|
| ○ No | |
| O Not enoug | gh information was given. |
| O None of th | ne above. |
| RESULTS B | x: |
| Correct! | |

Yes, the statement $\sim(x \lor y) \leftrightarrow (\sim x \land \sim y)$ is a tautology since its truth values are {T, T, T, T} as shown in the truth table below.

| x | у | ~x | ~y | x∨y | $\sim (x \lor y)$ | ~x^~y | $\sim (x \lor y) \leftrightarrow (\sim x \land \sim y)$ |
|---|---|----|----|-----|-------------------|-------|---|
| Т | Т | F | F | Т | F | F | Т |
| Т | F | F | Т | Т | F | F | Т |
| F | Т | Т | F | Т | F | F | Т |
| F | F | Т | Т | F | Т | Т | Т |

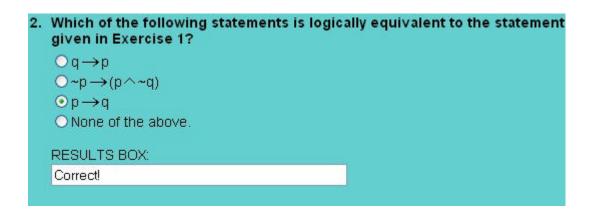


A conjunction is true when both parts are true. Since a statement and its negation have opposite truth values, the conjunction of a statement and its negation could never be true.

Lesson on Equivalence 1. What are the truth values of the following statement? $(p \land \neg q) \rightarrow \neg p$ $\bigcirc \{T, T, T, F\}$ $\bigcirc \{T, F, T, T\}$ $\bigcirc \{F, T, T, T\}$ $\bigcirc None of the above.$ RESULTS BOX: Correct!

The truth values of $(p \land \neg q) \rightarrow \neg p$ are $\{T, F, T, T\}$ as shown in the truth table below.

| | | | | p^~q | $(p^{a} \rightarrow q) \rightarrow \sim p$ |
|---|---|---|---|------|--|
| Т | Т | F | F | F | Т |
| Т | F | F | Т | Т | F |
| F | Т | Т | F | F | Т |
| F | F | Т | Т | F | Т |



The statement $p \rightarrow q$ is logically equivalent to the statement $(p \land \neg q) \rightarrow \neg p$, since they both have the same truth values, as shown in the truth table below.

| | q | p→q | $(p^{a} \rightarrow q) \rightarrow p$ |
|---|---|-----|---------------------------------------|
| Т | Т | Т | Т |
| Τ | F | F | F |
| F | Т | Т | Т |
| F | F | Т | Т |

| Which of the following statements is logically equivalent to q $ ightarrow$ (p $ ightarrow$ q)? |
|---|
| ⊙q→p |
| $\bigcirc \sim p \rightarrow (p \land \sim q)$ |
| $\bigcirc p \rightarrow q$ |
| O None of the above. |
| RESULTS BOX: |
| Correct |

The statement $q \rightarrow p$ is logically equivalent to the statement $q \rightarrow (p \land q)$ since they both have the same truth values, as shown in the truth table below.

| p | q | p^q | $q \rightarrow (p \land q)$ | q→p |
|---|---|-----|-----------------------------|-----|
| Т | Т | Т | Т | Т |
| Τ | F | F | Т | Т |
| F | Т | F | F | F |
| F | F | F | Т | Т |

| 4. | Which of the following statements is logically equivalent to a $ ightarrow$ (a $^{ ightarrow}$ b)? |
|----|--|
| | ⊙a→b |
| | O(a∨b)→b |
| | ⊙ (a^b) →b |
| | O None of the above. |
| | RESULTS BOX: |
| | Correct |
| | |

The statement $(a \land b) \rightarrow b$ is logically equivalent to the statement $a \rightarrow (a \lor b)$ since they both have the same truth values, as shown in the truth table below.

| a | b | a∨b | a^b | (a∧b)→b | a→(a∨b) |
|---|---|-----|-----|---------|---------|
| Т | Т | Т | Т | Т | Т |
| Т | F | Т | F | Т | Т |
| F | Т | Т | F | Т | Т |
| F | F | F | F | Т | Т |

| Given: | Statement x is logically equivalent to statement y. |
|-------------------------------|---|
| Problem: | Which of the following is true? |
| Ox if and | only if y |
| | s a tautology |
| Ox iff y | |
| All of th | e above. |
| RESULTS | BOX: |
| Correct! | |

Equivalent statements have the same truth values. Therefore, x and y satisfy the definition of a biconditional.. Thus, the statements listed in choice 1 and choice 3 are true. The biconditional of two equivalent statements is a tautology. Therefore, the statement listed in choice 2 is true.

Practice Exercises

| p | q | ~p | p ∩ q | p∨q | $p \rightarrow q$ |
|---|---|----|-------|-----|-------------------|
| Ţ | Т | F |] т | Т | Т |
| Т | F | F | F | Т | F |
| F | Т | Т | F | Т | Т |
| F | F | Т | E | F | Т |

This truth table shows the truth values for the negation of p, and for the conjunction, disjunction and conditional of statements p and q.

| | р | q | ~q | p ∧q | (p∧q)→~q |
|---|----|-----|----|-------------|----------|
| | Т |] т | F | Т | F |
| | °T | F | Т | F | T |
| | F | т | F | F | Т |
| Î | F | F | T | F | T |

This truth table shows the truth values for the compound statement $(p \land q) \rightarrow \sim q$.

| x | у | x→y | $y \rightarrow x$ | $(x \rightarrow y) \land (y \rightarrow x)$ | x↔y |
|---|---|-----|-------------------|---|-----|
| Т | Т | T | Т | Т | Т |
| Т | F | F | Т | F | F |
| F | T | Т | F | F | F |
| F | F | T | т | Т | Т |

This truth table shows the truth values of various compound statements involving x and y.

| Ox | | |
|-------------|----------|--|
| Oy | | |
| ⊙x↔y | | |
| None of the | e above. | |
| RESULTS BO | X: | |
| Correct | | |

The conditional statements in problem 3 are $x \rightarrow y$ and $y \rightarrow x$.

| $\bigcirc x \rightarrow y$ | | | |
|----------------------------|--------------|--|--|
| $\bigcirc y \rightarrow x$ | | | |
| ⊙x↔y | | | |
| ○ None c | f the above. | | |
| RESULTS | BOX: | | |
| Correct! | | | |

The biconditional statement from problem 3 is $x \leftrightarrow y$.

6.

| a | b | a→b | (a→b)∧a | $[(a \rightarrow b) \land a] \rightarrow b$ |
|---|---|-----|---------|---|
| Т | Т |] т | Т | Т |
| Т | F | F | F | Т |
| F | T | Т | F | т |
| F | F | т | F | Т |

This truth table shows the truth values of various compound statements involving a and b.

| OBiconditional | |
|-------------------------------|--|
| Tautology | |
| O Disjunction | |
| ○None of the above. | |
| RESULTS BOX: | |
| Correctl | |

The statement in the last column of the truth table in problem 6 is a tautology since all of its truth values are true.

| р | q | ~q | p→~d | p∧q | ~(p ^ q) | $(p \rightarrow \neg q) \leftrightarrow [\neg (p \land q)]$ |
|---|---|----|------|-----|----------|---|
| Т | Т | F | F | Т | F | Т |
| Т | F | T | Т | F | T | Т |
| F | Т | Т | Т | F | Т | Т |
| F | F | T | Т | F | т | Т |

The truth values for the last column are all true. Thus the statement $(p \rightarrow \sim q) \leftrightarrow [\sim (p \land q)]$ is a tautology.

| $\bigcirc p \rightarrow \sim q$ and | p^q |
|--|----------|
| \bigcirc p \land q and \sim () | |
| $\odot p \rightarrow \sim q$ and \cdot | ~(p ^ q) |
| ○ None of the a | bove. |
| RESULTS BOX: | |
| Correct | |

The statements $p \rightarrow \sim q$ and $\sim (p \land q)$ have the same truth value. These statements are, therefore, logically equivalent.

| The | that best completes this sentence: of two equivalent statements always yields a tautology |
|------------------------|--|
| Biconditional | |
| O Conjunction | |
| O Negation | |
| ○ All of the abov | /e. |
| RESULTS BOX: | |
| Correct! | |

The biconditional of two equivalent statements is a tautology.