SILESIAN UNIVERSITY
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Mathematics in economics

Lecture 6

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Mathematics in Economics/PMAT

Integration of rational functions – continued
Problem 2

Solve: $\int \frac{6x^3 \ldots + \ldots}{\ldots}$.

Solution: It is a case 2, the root x = -1 is of the order 3.

Partial fraction decomposition:

$$\frac{6x^3 + \quad + \quad +}{x + \ldots} \quad \frac{B}{+} \quad \frac{C}{+} \quad D$$

After a rearrangment we yield:

$$\frac{6x^3 + \quad + \quad +}{x + \ldots} \quad + \quad + \quad + \quad + \quad +$$

Integration of rational functions
Problem 2 - cont

By clearing the denominator and solving the equation:
A = 1, B = 4, C = -2, D = 5.

The division:

$$\int \quad\quad\quad \int \quad\quad \int \quad\quad \int \quad\quad \int$$

[illegible]

And integration:

$$= \quad + - \quad + \quad + \quad + $$

[illegible]

Integration of rational functions
Problem 3

Solve: $\int 3x^2$.

Solution: It is the case 3. Therefore, the partial fraction decomposition is:

$$\frac{3x^2}{\ldots} - + $$

Solving the equality of numerators yields:

$$\int 3x^2 \quad \int \quad \int$$

And, finally: $\int 3x^2 \quad = \quad + \quad 1 \quad + -$

Integration per partes

Per partes method (integration by parts) is used for integration of a product of two functions.
Let u(x) and v(x) be two functions. Then, we obtain:

$$u \cdot \quad = \quad +$$

$$uv \quad = \quad -$$

$$\int \quad \int \quad \int$$

$$\int \quad \int$$

The last formula is “per partes“ formula.

Integration per partes - Problem 1

Solve: $\int$.

Solution:

$\int \qquad \int \qquad - \quad +$

Note: a choice of u and v is important. An incorrect choice leads to a growing difficulty of a problem.

Integration per partes - Problem 2

Solve: $\int$.

Solution:

$$\int \ln x\, d = \left| \begin{array}{l} u = \ln x, v' = \\ = 1, v = \frac{x^2}{2} \end{array} \right| = \frac{x^2}{2}\ln x - \int \frac{1}{\cdot}\,\frac{x^2}{2}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int d = \frac{x^2}{2}\ln x - \frac{x^2}{4} +$$

Note: a choice of u and v is important. An incorrect choice (v = lnx) leads to a growing difficulty of a problem.

Integration per partes - Problem 3

Solve: $\int$.

Solution:

$$\int \sin x\,dx = \left| \begin{matrix} u = , & v' = \sin x \\ u' = 1, & v = -\cos x \end{matrix} \right| = \cos x - \int (-\cos x)\,dx = \cos x + \sin x +$$

Note: a choice of u and v is important. An incorrect choice (v = x) leads to a growing difficulty of a problem.

Integration per partes - Problem 4

Solve: $\int arctgx$.

Solution: we will use a trick – let u = arctgx and v = 1:

$$\int arctgx = \left| \begin{matrix} u = arctgx & u' = \frac{1}{1+x^2} \\ v' = 1 & v = x \end{matrix} \right| = x\,arctgx - \int \frac{x}{1+x^2}dx = x\,arctgx - \frac{1}{2}\ln\left|x^2+1\right| + C$$

Problems to solve 1 (Assignment 8)

Find:

$$\int \frac{x-13}{x^2+4x-5}dx$$

$$\int \frac{5x^2-17x+12}{x^3-4x^2+4x}dx$$

$$\int$$

$$\int$$

Integration by a substitution

We use a substitution typically in the following cases:

- When an integrand contains an internal function.
- When na integrand contains lnx or exp(x).
- When an integrand contains goniometric functions.
- When an integrand contains square roots.

Integration by a substitution – Problem 1

Find: $\int$

Solution:

$$\int \qquad \int \quad \int \qquad - = \qquad +$$

A note: We substitute not only an integrand, but also dx!

Integration by a substitution – Problem 2 and 3

Find: $\int$

Solution: $\int$ 2x $\int$ dt 1 $\int$ 1 1 $+$

Find: $\int$

Solution:

$\int$ 5x $\int$ dt 1 $\int$ sin t sin(5x $+$

Integration by a substitution – Problem 4 and 5

Find: $\int \frac{\ln x}{}$

Solution: $\int \frac{\ln x}{} \quad \ln x \quad \int \quad \frac{t^2}{} \quad \frac{\ln^2 x}{}$

Find: $\int$

Solution: $\int \frac{5}{\,\ln x}dx = \quad \ln x = \quad \frac{1}{x}dx = \quad = \int \frac{5}{}dt = 5 \quad + \quad 5 \quad = \ln \quad +$

Integration by a substitution – Problem 6 and 7

Find: $\int e^{x}$

Solution:

$$\int \frac{e^x}{2e^x+5}dx = \dots = \int \frac{1}{2t+5}dt = \frac{1}{2}\ln|2t+5| + \dots = \frac{1}{2}\ln|2e^x+5| + \dots$$

Find: $\int e^{x}$

Solution:

$$\int e^{x} \dots \int \dots \int \dots \int \dots + \dots$$

Integration of goniometric functions

Useful identities:

(1)	$\sin^2 x + \cos^2 x = 1$
(2)	$\sin 2x = 2\sin x \cdot \cos x$
(3)	$\cos 2x = \cos^2 x - \sin^2 x$
(4)	$\sin^2 x = \dfrac{1-\cos 2x}{2}$
(5)	$\cos^2 x = \dfrac{1+\cos 2x}{2}$

A universal goniometric substitution

(1)	$tg\frac{x}{2}=t$
(2)	$\sin x=\frac{2t}{t^2+1}$
(3)	$\cos x=\frac{1-t^2}{t^2+1}$
(4)	$dx=\frac{2}{1+t^2}$

Integration of goniometric functions - Problems 1 and 2

Find: $\int$

Solution:

$$\int \quad \sin x \qquad \int \quad 4 \qquad \sin 4x$$

Find: $\int$

Solution:

$$\int \quad 1 \qquad \int \qquad \int \quad 1 \qquad x \quad \sin 2x \quad +$$

Integration of goniometric functions - Problems 3 and 4

Find: $\int$

Solution:

Find: $\int$

Solution:

Integration of irrational functions – Problem 1

Usually, we substitute (square roots).

Find: $\int$.

Solution:

$$\int \quad \left| \sqrt{4x} + = \right. \quad \int \quad \int \quad = \quad \sqrt{4x} \quad +$$

Integration of irrational functions – Problem 2

Find: $\int$

Solution:

$$\left|\sqrt{x^2} - = \right.$$

$$\int \qquad \int \qquad \int \qquad t^3 \qquad \sqrt{x^2} \;\; ^3 \qquad +$$

$$\left| x\,dx = \right.$$

A note: see also Euler s subtitutions.

Integration of irrational functions – Problem 3

Find: $\int$

Solution: in the process of integration, we use goniometric Substitution as well!

$\int \quad \int \quad \int \quad \int$

$= \int \quad + \quad = \quad + \quad - \quad +$

Problems to solve - 1

Find:

$\int$

$\int$

$\ln^2 x$

$\int$

$\int$

$\int$

Problems to solve - 2

Find:

$\int$

$\int$

$\int$

$\int$

Thank you for your attention!