**Mathematics in Economics – lecture 3**

1. Composite derivative (Chain rule)

In simple words, we say that the derivative of a composite function is **the product of the derivative of the outside function with respect to the inside function and the derivative of the inside function with respect to the variable**.



 $y=\left(x^{3}+4x^{2}\right)^{5}$



1. The second derivative

The derivative of a function y = f(x) of a variable x is **a measure of the rate at which the value y of the function changes with respect to the change of the variable x**. It is called the derivative of f with respect to x.

The second derivative is **the rate of change of the rate of change of a point at a graph** (the "slope of the slope" if you will). This can be used to find the acceleration of an object (velocity is given by first derivative).

If a function f´(x) can be differentiated, we obtain the second derivative of f(x), denoted as f´´(x), and so on.

1. $y= 3x^{4}+2x^{2}-x+1$ Find $y^{´´}\left(2\right)=$
2. $y=4x^{3}+5x+1$ Find $y^{´´´}\left(1\right)=$
3. $y=-5x^{4}+3x^{3}+1$ Find $y^{´´´}\left(0\right)=$
4. Taylor and Maclaurin series

The **Taylor series** of a [function](https://en.wikipedia.org/wiki/Function_%28mathematics%29) is an [infinite sum](https://en.wikipedia.org/wiki/Series_%28mathematics%29) of terms that are expressed in terms of the function's [derivatives](https://en.wikipedia.org/wiki/Derivative) at a single point. For most common functions, the function and the sum of its Taylor series are equal near this point. Taylor series are named after [Brook Taylor](https://en.wikipedia.org/wiki/Brook_Taylor), who introduced them in 1715.

If 0 is the point where the derivatives are considered, a Taylor series is also called a **Maclaurin series**, after [Colin Maclaurin](https://en.wikipedia.org/wiki/Colin_Maclaurin), who made extensive use of this special case of Taylor series in the 18th century.

Let a function *y* = *f*(*x*) be differentiable of the order *n* at a point *a*, then it can be approximated by the Taylor series of the form:



* If *a* = 0, we obtain a special case of the Taylor series, called Maclaurin series:



**Maclaurin series of selected functions**

|  |  |
| --- | --- |
| **Function** | **Maclaurin series** |
| **sinx** | $$x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}+...$$ |
| **cosx** | $$1-\frac{x^{2}}{2!}+\frac{x^{4}}{4!}-\frac{x^{6}}{6!}+...$$ |
| **exp(x)** | $$1+\frac{x}{1!}+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+...$$ |

1. Find the Taylor series of the function $f\left(x\right)=3x^{3}+2x^{2}-10x+2$ at the point $a=2$.
2. Find the Maclaurin series of the function $f\left(x\right)=2x^{4}+3x^{2}-6x+3$. a=0