**Mathematics in Economics – lecture 6**

**Extremes of a function of two real variables**

Necessary condition for the extreme



A point satisfying equalities above is called a stationary (critical) point. However, this condition is not sufficient.

In a critical point can be maximum, minimum or an inflection point. To decide which situation occurs, we use the second derivatives and a matrix called *hessian* H(*C*)



**Determinant calculation:**

we multiply the numbers on the main diagonal and subtract the product of the numbers on the secondary diagonal.

Then we use Sylvester´s theorem.

We denote: D1 = $f´´\_{xx}\left(C\right)$ and D2 = H(*C*). Then:

If D2>0, then we have an extreme.

Moreover, If D1>0, we have a minimum, if D1<0, we have a maximum.

IF D2<0, we have an inflection point. If D2 = 0, we cannot decide.

**Problem 1**

Find extremes of the function $f\left(x,y\right)=x^{3}-2xy$

***Solution:*** We start with the first derivatives:

Both derivatives must be 0, which yields the critical point C [0,0].

Now we compute all second derivatives and hessian:

We substitute point C into hessian: *Hf*(0,0) =

 Because D2<0, the point C is an inflection point.

**Problem 2**

Find extremes of the function $f\left(x,y\right)=x^{2}-2xy+y$ .

***Solution:*** We start with the first derivatives:

Both derivatives must be 0, which yields the critical point *C* [1/2, 1/2].

Now we compute all second derivatives and hessian:

We substitute point C into hessian: *Hf*(1/2; 1/2) =

 .

Because D2<0, the point C is an inflection point.

**Problem 3**

Find extremes of the function $f\left(x,y\right)=-3x^{2}+2xy-2y^{2}-10$ .

***Solution:*** We start with the first derivatives:

Both derivatives must be 0, which yields the critical point *C* [0, 0].

Now we compute all second derivatives and hessian:

We substitute point C into hessian: *Hf*(0; 0) =

 .

Because D2 > 0, we have extreme at the point C; because D1 < 0, we have a maximum.

**Problem 4**

Find extremes of the function $f\left(x,y\right)=x^{2}+4xy+6y^{2}-2x+8y-5$ .

***Solution:*** We start with the first derivatives:

Both derivatives must be 0, which yields the critical point *C* [7, –3].

Now we compute all second derivatives and hessian:

We substitute point C into hessian: *Hf*(7; –3) = .

Because D2 > 0, we have extreme at the point C; because D1 > 0, we have a minimum.

**Problem 5**

Find the maximum of the revenue function: $TR\left(Q\_{1},Q\_{2}\right)=50Q\_{1}+20Q\_{2}-2Q\_{1}^{2}-5Q\_{2}^{2}$

***Solution:*** We start with the first derivatives:

Both derivatives must be 0, which yields the critical point *C* [12.5 , 2].

Now we compute all second derivatives and hessian:

We substitute point C into hessian: *Hf*(12.5; 2) = .

Because D2 > 0, we have extreme at the point C; because D1 < 0, we have a maximum.

**HOMEWORK**

A] 

B] 

C] 

D] 