# Quantitative Methods

## Lecture 1

Introduction,
sets and mathematical language

SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

INM/BAKVM

# Outline of the lecture

**Requirements**

1) 70% attendance at the seminars (or calculating a mathematical problem or writing a seminar paper)

2) Two tests

a) for 30 points=TEST (on the 6th of November) and

b) for 70 points= FINAL EXAM (on the 11th of December).

Form of the exam: written. You can gain extra point for tasks and homework.

Evaluation: A (100-90 points), B (89-80), C (79-70), D (69-65), E (64-60), F (59-0).

## Syllabus (short version)

- 1. Motivational introduction, history of mathematics
- 2. Algebraic Expressions
- 3. Equations and Inequalities
- 4. Matrix calculus
- 5. Determinants
- 6. Systems of linear algebraic equations
- 7. Sequences, limits of sequences
- 8. Basic functions of one real variable
- 9. Limits of functions of one real variable
- 10. Differential calculus of functions of one real variable
- 11. Using differential calculus of functions of one real variable
- 12. Integral calculus of functions of one variable and its applications
- 13. Application of differential and integral calculus in economics and management

# Outline of the lecture

- Mathematical language ($\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$)
- Sets and set operations ($\in, \cap, \cup, \setminus$)
- Mathematical language: Quantifiers ($\forall, \exists$)
- Set inclusion ($\subseteq$)
- More on sets
- Number domains ($\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$) (set of natural numbers, set of integers, set of rational numbers, set of irrational numbers, set of complex numbers)

# Mathematical language

- Simple propositions
- Logical conjunctions
- Some useful equivalences
- Sylogism

# Mathematical language

A **simple proposition** is any statement such that it makes sense to say whether it is true or false.

Examples of simple propositions:
— "It is raining."
— "Today is Saturday."
— $2 + 2 = 4$

The following *are not* simple propositions:
— "Are you happy?"
— "Come here!"

# Mathematical language

Let $A$ and $B$ be simple propositions.

We can join them into **compound propositions**, or mathematical sentences, by using logical conjunctions.

There are
— up to 4 distinct unary logical conjunctions (such as $\neg$) and
— up to 16 distinct binary logical conjunctions (such as $\wedge, \vee, \Rightarrow, \Leftrightarrow$).

We shall mention some of them.

# Mathematical language

Logical **negation:** read "$\neg A$" as "**not** $A$"

Table of truth values:

| truth value of given simple proposition | result |
|---|---|
| $A$ | $\neg A$ |
| false | true |
| true | false |

# Mathematical language

**Logical conjunction:** read “$A \wedge B$” as “$A$ and $B$”

Table of truth values:

| truth values of given simple propositions | | result |
|---|---|---|
| $A$ | $B$ | $A \wedge B$ |
| false | false | false |
| false | true | false |
| true | false | false |
| true | true | true |

# Mathematical language

**Logical equivalence:** read "$A \Leftrightarrow B$" as "$A$ **if and only if** $B$"

Table of truth values:

| truth values of given simple propositions | | result |
|---|---|---|
| $A$ | $B$ | $A \Leftrightarrow B$ |
| false | false | true |
| false | true | false |
| true | false | false |
| true | true | true |

# Mathematical language

**Logical implication:** read "$A \Rightarrow B$" as "if $A$, then $B$"

Table of truth values:

| truth values of given simple propositions | | result |
|---|---|---|
| $A$ | $B$ | $A \Rightarrow B$ |
| false | false | true |
| false | true | true |
| true | false | false |
| true | true | true |

# Some equivalences I

Let $A$ and $B$ be simple propositions.

Notice and observe that the following equivalences are easy to see:

Identity:

$$A \Leftrightarrow A$$

Idempotency:

$$(A \wedge A) \Leftrightarrow A$$

$$(A \vee A) \Leftrightarrow A$$

# Some equivalences II

Let $A$ and $B$ be simple propositions.

Notice and observe that the following equivalences are easy to see:

Double negation:

$$A \Leftrightarrow \neg\neg A$$

*Tertium non datur*:

$$A \vee \neg A$$

# Some equivalences III

Let $A$ and $B$ be simple propositions.

Notice and observe that the following equivalences are easy to see:

De Morgan's Laws:

$$\neg(A \wedge B) \Leftrightarrow (\neg A \vee \neg B)$$

$$\neg(A \vee B) \Leftrightarrow (\neg A \wedge \neg B)$$

# Some equivalences IV

Let $A$ and $B$ be simple propositions.

Notice and observe that the following equivalences are easy to see:

Commutativity:

$$(A \wedge B) \Leftrightarrow (B \wedge A)$$

$$(A \vee B) \Leftrightarrow (B \vee A)$$

Associativity:

$$A \wedge (B \wedge C) \Leftrightarrow (A \wedge B) \wedge C$$

$$A \vee (B \vee C) \Leftrightarrow (A \vee B) \vee C$$

# Sylogism

Let $A, B, C$ be simple propositions.

Notice and observe that the following proposition is easy to see:

**Sylogism:**

$$(A \Rightarrow B) \Longrightarrow \big((B \Rightarrow C) \Rightarrow (A \Rightarrow C)\big)$$

Example:
- Socrates is a man.
- A man is mortal.

Conclude that:
- Socrates is mortal.

By using the "import" rule:

$$\big((A \Rightarrow B) \land (B \Rightarrow C)\big) \Longrightarrow (A \Rightarrow C)$$

In the example:

$A$ = Socrates
$B$ = a man
$C$ = mortal

# Sets and set operations

- Sets
- The empty set
- Set operations
- Some useful equalities

# Sets

A **set** is a collection of **elements**.

Sets are denoted by upper-case letters: $A, B, C, \ldots$

Elements are denoted by lower-case letters: $a, b, c, \ldots, x, y, z, \ldots$

Let a set $A$ be given. Then, for any element $x$,
we must be able to say whether
— either the element $x$ is in the set $A$, or
— or the element $x$ is not in the set $A$.

Notice that we do not allow any third possibility.

# Sets

Let a set $A$ and an element $x$ be given.

We write the fact that the **element** $x$ **is in the set** $A$ as

$$x \in A$$

We write the fact that the **element** $x$ **is not in the set** $A$ as

$$x \notin A$$

# Sets

A set is often given by the list of its elements. For example:

$$A = \{1, 3, 5, 7, 9\}$$
$$B = \{2, 4, 6, 8\}$$

Then, for example, it holds

$$1 \in A, \qquad 2 \in B, \qquad 1 \notin B, \qquad 2 \notin A$$

# The empty set

The **empty set** is a set that contains no elements:

$$\emptyset = \{\,\}$$

Notice that the set $\{\emptyset\}$ is **not** empty, we have

$$\emptyset \in \{\emptyset\}$$

and

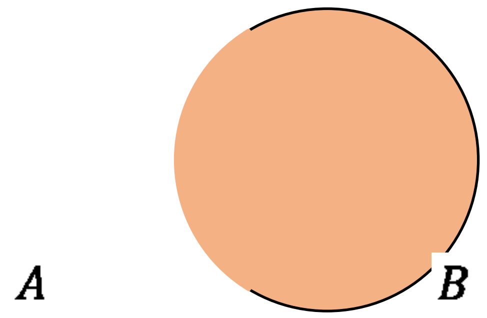
$$\{\emptyset\} \neq \emptyset$$

# Set operations

Let $A$ and $B$ be sets.

The **union** of the sets is:

$$A \cup B = \{x : x \in A \lor x \in B\}$$

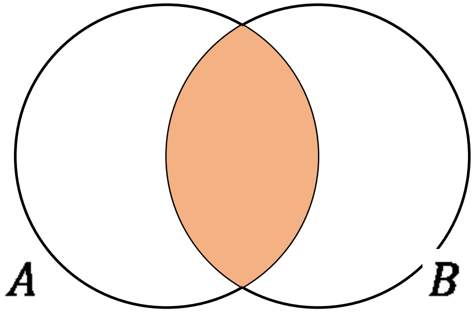


# Set operations

Let $A$ and $B$ be sets.

The **intersection** of the sets is:

$$A \cap B = \{x : x \in A \land x \in B\}$$

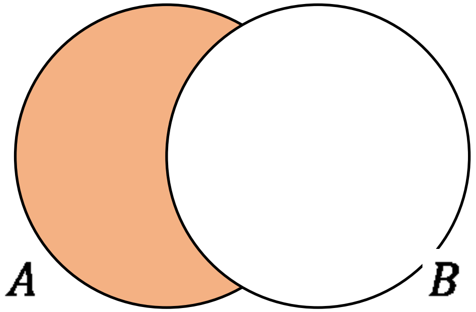


# Set operations

Let $A$ and $B$ be sets.

The **difference** of the sets is:

$$A \setminus B = \{x : x \in A \land x \notin B\}$$



# Set inclusion

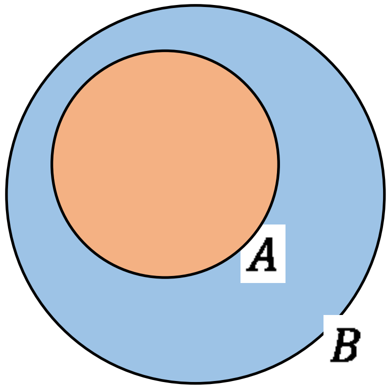
Let $A$ and $B$ be sets.

We say that $A$ **is a subset of** $B$ (or $A$ **is included in** $B$) and write

$$A \subseteq B$$

if and only if

$$\forall x: x \in A \Rightarrow x \in B$$



# Set operations

Let $A$ and $B$ be sets.

**Union:**

$$A \cup B = \{x : x \in A \vee x \in B\}$$

**Intersection:**

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

**Difference:**

$$A \setminus B = \{x : x \in A \wedge x \notin B\}$$

# Mathematical language: Quantifiers

Let $x$ be a variable and let $\varphi(x)$ be a (compound) proposition or mathematical sentence (containing $x$). Then

$$\forall x: \varphi(x)$$

reads

"for all $x$, it holds $\varphi(x)$"

Let $A$ be a set, let $x$ be a variable, and let $\varphi(x)$ be a proposition. Then

$$\forall x \in A: \varphi(x)$$

reads

"for each element $x$ of the set $A$, it holds $\varphi(x)$"

# Mathematical language: Quantifiers

Let $x$ be a variable and let $\varphi(x)$ be a (compound) proposition or mathematical sentence (containing $x$). Then

$$\exists x: \varphi(x)$$

reads

**"there exists an $x$ such that it holds $\varphi(x)$"**
**("there exists at least one $x$ such that it holds $\varphi(x)$")**

Let $A$ be a set, let $x$ be a variable, and let $\varphi(x)$ be a proposition. Then

$$\exists x \in A: \varphi(x)$$

reads

**"there exists at least one element $x$ of the set $A$ such that it holds $\varphi(x)$"**

# De Morgan's Laws I

Let $x$ be a variable and let $\varphi(x)$ be a proposition or mathematical sentence.

Notice and observe that the following equivalences are easy to see:

De Morgan's Laws:

$$\neg\forall x\colon \varphi(x) \Leftrightarrow \exists x\colon \neg\varphi(x)$$

$$\neg\exists x\colon \varphi(x) \Leftrightarrow \forall x\colon \neg\varphi(x)$$

## Number domains

In mathematics, we use the following number sets:

**Natural** numbers:

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$
$$\mathbb{N}_0 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

**Integer** numbers:

$$\mathbb{Z} = \{\dots, -3, -2, -2, -1, 0, 1, 2, 3, \dots\}$$

**Rational** numbers:

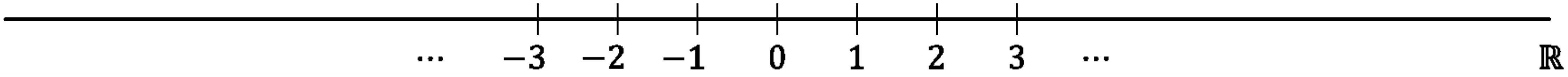
$$\mathbb{Q} = \left\{\frac{p}{q} : p, q \in \mathbb{Z} \wedge q \neq 0\right\}$$

# Number domains

**Real numbers:**

$$\mathbb{R} = \cdots$$

the definition is rather difficult, but the real numbers can be depicted as the points of a line:



# Number domains

**Complex numbers:**

$$\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$$

where "i" is the imaginary unit ($i^2 = -1$).