Economic applications

Economic applications of derivatives

The elasticity of a function y = f(x):

$$E(x) = \lim_{\Delta x \to 0} \frac{x}{y} \frac{\Delta y}{\Delta x} = \frac{x}{y} \frac{dy}{dx} = \frac{x}{y} y'$$

The price elasticity of demand:

$$E(P) = -\frac{P}{Q} \frac{dQ}{dP}$$

The price elasticity of supply:

$$E(P) = \frac{P}{Q} \frac{dQ}{dP}$$

Marginal product of labour:

$$MP_L = \frac{dQ}{dL} = Q'(L)$$

Marginal revenue:

$$MR = \frac{dTR(Q)}{dQ}$$

Marginal cost:

$$MC = \frac{dTC}{dQ}$$

Solved problems

Find marginal revenue MR (x) of the total revenue

$$TR(x) = x^3 - 2x^2 + 5x + 5$$

and marginal costs of the total costs

$$TC(x) = 120x^4 - \ln x$$

Solution:

$$MR(x) = \frac{dTR(x)}{dx} = 3x^2 - 4x + 5$$

$$MC(x) = \frac{dTC(x)}{dx} = 480x^3 - \frac{1}{x}$$

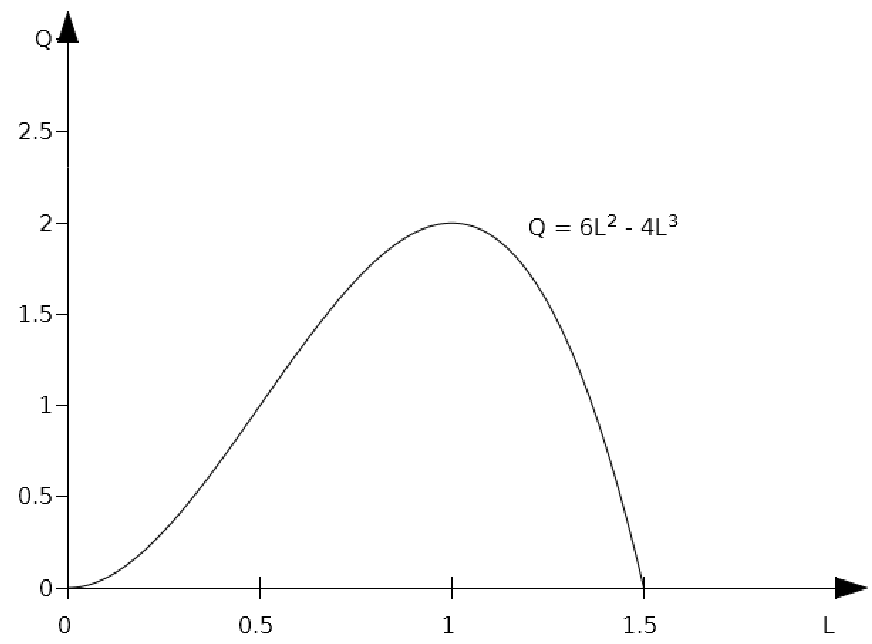
Solved problem

Find extremes of the production function $Q = 6L^2 - 4L^3$. Draw its graph.

Solution:

The first derivative is $Q' = 12L - 12L^2$, we find roots of the first derivative: L = 0 and L = 1. By the use of the second derivative, or by checking signs of the first derivative, we obtain that L = 0 is a local minimum and L = 1 is a local maximum. Therefore, the highest (optimal) production is achieved when L = 1.

The graph is provided on the next slide.


Q
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
L
Q = 6L² - 4L³

Assignment

Find the maximum of total revenue function $TR(Q) = -1400 + 80Q - Q^2$.

Find the minimum of total cost function: $TC(Q) = 100 - 60Q + Q^2$.

Find the maximum of the profit function: $PR(Q) = 100 + 64Q - 4Q^2$.

Find the maximum of total revenue function: $TR(Q) = -80Q^2 + 160Q + 200$.

Differential calculus of two real variables

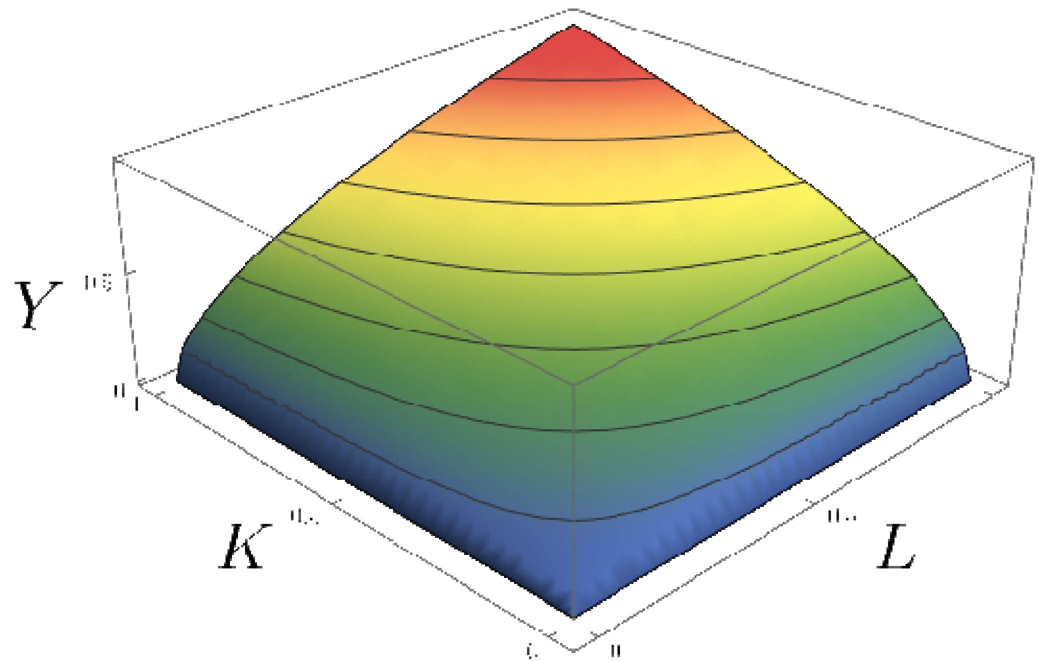
Many economic functions contain more then one variable.

For example, **Cobb-Douglas function** includes labour L and capital K as well as the technological parameter A:

$$Q(K,L) = AK^{\alpha}L^{\beta}$$

We will limit ourselved to functions of two variables. A graph of a function of two real variables is a plane in 3D space,
See the next slide.

A graph of Cobb-Douglas function



Cobb – Douglas function

C-D function: $Q = AK^{a} \cdot L^{b}$

Usually, we assume that $a + b. = 1$

Then, C-D: $Q = AK^{a} L^{1-a}$

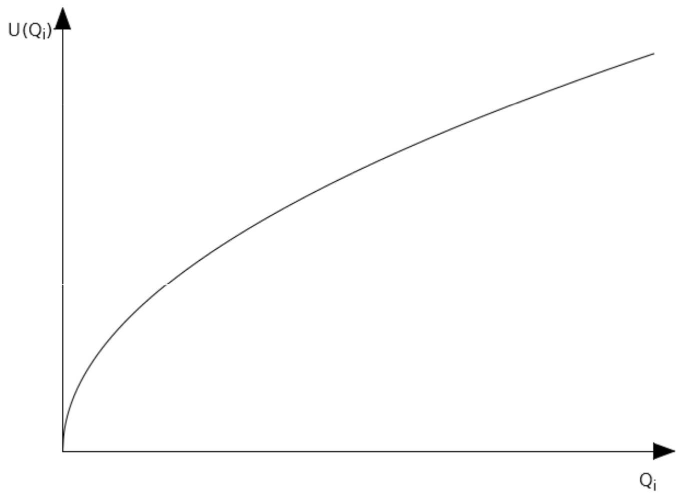
Marginal product of labour: $MP_L = \frac{\partial Q}{\partial L} = AK^{a}(1-a)L^{-a} = \frac{A}{1-a}\left(\frac{K}{L}\right)^{a}$

Marginal product of capital: $MP_K = \frac{\partial Q}{\partial K} = AaK^{a-1}L^{1-a} = Aa\left(\frac{K}{L}\right)^{a-1}$

A utility function

Let n be the number of different types of good. Let Q_1, Q_2, …. Be the amount of the good 1, 2, etc.

Then a function $U(Q_1, Q_2, \ldots, Q_n)$ is called the utility function. Typically, a utility function is concave:



Marginal utility

Marginal utility is defined as follows:

$$MU_1 = \frac{\partial U(Q_1, Q_2)}{\partial Q_1} \qquad MU_2 = \frac{\partial U(Q_1, Q_2)}{\partial Q_2}$$ etc.

Example:

Find marginal utilities of the utility function $U = Q_1^{0,5} \cdot Q_1^{0,2}$

Solution:

$$MU_1 = \frac{\partial U}{\partial Q_1} = 0,5Q_1^{-0,5} \cdot Q_1^{0,2}$$

$$MU_2 = \frac{\partial U}{\partial Q_2} = 0,2Q_1^{0,5} \cdot Q_1^{-0,8}$$