

Lesson 4

1. Calculate

a) $\begin{vmatrix} 3 & 1 \\ 2 & 5 \end{vmatrix}$

b) $\begin{vmatrix} 9 & -4 \\ 5 & 3 \end{vmatrix}$

c) $\begin{vmatrix} -2 & -1 \\ 8 & 4 \end{vmatrix}$

d) $\begin{vmatrix} 4 & 0 & -1 \\ 2 & 2 & 3 \\ -4 & 1 & 1 \end{vmatrix}$

e) $\begin{vmatrix} 2 & 0 & -1 \\ 4 & 1 & 2 \\ 6 & 1 & 1 \end{vmatrix}$

2. Solve inequalities:

a) $\begin{vmatrix} x+2 & -3 \\ 2x & 4 \end{vmatrix} \leq 2$

b) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2-x & 1 \\ 1 & 1 & 3+x \end{vmatrix} \geq 0$

3. Solve using Cramer's rule:

a) $\begin{cases} 2x - 3y = 5 \\ -x + 2y = -3 \end{cases}$

b) $\begin{cases} x + y + z = 1 \\ 2x - y + z = -2 \\ 4x + y + z = 4 \end{cases}$

c) $\begin{cases} x + y + z = 6 \\ 2x - 4y + z = -3 \\ 3x - y - z = -2 \end{cases}$

d) $\begin{cases} 2x + 3y + 2z = 2 \\ x + y + 2z = -1 \\ 2x + 4y = 6 \end{cases}$

e) $\begin{cases} x + y - z = 5 \\ 2x - y + z = 4 \\ 4x + y - z = 0 \end{cases}$

A matrix is in reduced **row-echelon form** if it satisfies the following: In each row, the left-most nonzero entry is 1 and the column that contains this 1 has all other entries equal to 0. This 1 is called a leading 1. The leading 1 in the

Row Echelon Form

Property 2 : If a column contains a leading entry then all entries below that leading entry are zero.

$$\begin{bmatrix} 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 & 7 \\ 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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Row Echelon Form (REF) Fundamentals

Two simple rules for REF:

1. All rows of zeros must be at the bottom
2. Staircase pattern of **first** non-zero entries in each row (non-zero entries in each row are to the right of the one above)

$$\begin{bmatrix} 2 & 5 & 3 \\ 0 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} \quad \checkmark \quad \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad \times$$