Disjunction

Example 1:

Given:	p: Ann is on the softball team.
	q: Paul is on the football team.
Problem:	What does p$\vee$q represent?

Solution: In Example 1, statement p represents, "Ann is on the softball team" and statement q represents, "Paul is on the football team." The symbol $\vee$ is a logical connector which means "or." Thus, the compound statement p$\vee$q represents the sentence, "Ann is on the softball team or Paul is on the football team." The statement p$\vee$q is a disjunction.

Definition: A **disjunction** is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by p$\vee$q. A disjunction is false if and only if both statements are false; otherwise it is true. The truth values of p$\vee$q are listed in the truth table below.

p	q	p$\vee$q
T	T	T
T	F	T
F	T	T
F	F	F

Example 2:

Given:	a: A square is a quadrilateral.
	b: Harrison Ford is an American actor.
Problem:	Construct a truth table for the disjunction "a or b."

Solution:

a	b	a$\vee$b
T	T	T
T	F	T
F	T	T
F	F	F

Example 3:

Given:	r: x is divisible by 2.
	s: x is divisible by 3.
Problem:	What are the truth values of r$\vee$s?

Sol∨on: Each statement given in this example represents an open sentence, so the truth value of r ∨ s will depend on the replacement values of x as shown below.

If x = 6, then r is true, and s is true. The disjunction r∨s is true.

If x = 8, then r is true, and s is false. The disjunction r∨s is true.

If x = 15, then r is false, and s is true. The disjunction r∨s is true.

If x = 11, then r is false, and s is false. The disjunction r∨s is false.

Example 4:

Given:	p: 12 is prime.	false
	q: 17 is prime.	true
	r: 19 is composite.	false
Problem:	Write a sentence for each disjunction below. Then indicate if it is true or false.	

1.	p∨q	12 is prime or 17 is prime.	true
2.	p∨r	12 is prime or 19 is composite.	false
3.	q∨r	17 is prime or 19 is composite.	true

Example 5: Complete a truth table for each disjunction below.

1. a or b

2. a or not b

3. not a or b

a	b	a∨b
T	T	T
T	F	T
F	T	T
F	F	F

a	b	~b	a∨~b
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

a	b	~a	~a∨b
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Students sometimes confuse conjunction and disjunction. Let's look at an example in which we compare the truth values of both of these compound statements.

Example 6:

Given:	x: Jayne played tennis.
	y: Chris played softball.
Problem:	Construct a truth table for conjunction "x and y" and disjunction "x or y."

Solution:

x	y	x∧y	x∨y
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

With a conjunction, **both statements must be true for the conjunction to be true**; but with a disjunction, **both statements must be false for the disjunction to be false**. One way to remember this is with the following mnemonic: 'And' points up to the sand on top of the beach, while 'or' points down to the ore deep in the ground.

Summary: A **disjunction** is a compound statement formed by joining two statements with the connector OR. The disjunction "p or q" is symbolized by p∨q. A disjunction is false if and only if both statements are false; otherwise it is true.

Exercises

Directions: Read each question below. Select your answer by clicking on its button. Feedback to your answer is provided in the RESULTS BOX. If you make a mistake, choose a different button.

1. Which of the following sentences is a disjunction?

- Amy played soccer or Bill played hockey.
- Amy played soccer and Bill played hockey.
- Amy did not play soccer and Bill played hockey.
- None of the above.

RESULTS BOX:

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2. Which of the following statements is a disjunction?

- ~x∧y
- x∧y

- x ∨ y
- None of the above.

RESULTS BOX:

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3. A disjunction is used with which connector?

- And
- Or
- Not
- None of the above.

RESULTS BOX:

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4. If a is false and b is true, what is the truth value of a ∨ ~b?

- True
- False
- Not enough information was given
- None of the above.

RESULTS BOX:

5.

Given:	**r: y is prime.**
	s: y is even.
Problem:	**Which of the following is a true statement when y is replaced by 3?**

- r ∨ ~s
- r ∧ ~s
- r ∨ s
- All of the above.

RESULTS BOX: