SILESIAN UNIVERSITY
SCHOOL OF BUSINESS ADMINISTRATION IN KARVINA

Mathematics in economics

Lecture 10

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Geometric function series

Geometric function series is defined as follows:

$$\sum$$

The series is convergent if $|q| <$, where q = f(x).

The sum is given as: $s(x) =$

Geometric function series – Problem 1

Find the range of convergence: $\sum$.

Solution:

Expanding the sum yields: $\sum \quad + \quad + \quad +$

Clearly, $a_1 = q = x$.

Because $|q| <$ we have $|x| <$. The range of convergence:

$$\mathcal{K} = -$$

Geometric function series – Problem 2

Find the sum of the series: $\sum$.

Solution:
We already know that $a_1 = q = x$.

Using the formula for the sum yields: $s(x) = \frac{x}{\;}$

This result is valid for all x satysfying $|x| <$

Geometric function series – Problem 3

Find the range of convergence and a sum of the series:

$$\sum$$

Solution: $a_1 =$, $q =$

The convergence: $|q| =$

Which yields: $CK = -\infty \quad \cup \quad \infty$

The sum: $s(x) =$

Problems to solve

Find the range of convergence and a sum of the series:

$$\sum$$

$$\sum$$

$$\sum_{n=1}^{\infty} n x^n$$

$$\sum$$

$$\sum_{n=1}^{\infty} \frac{1}{n+1} \left(\frac{x}{2} \right)^n$$

Differential equations

Differential equation (DE) is an equation that includes given function y = f(x) and its derivatives.

Examples:

$y' + \quad =$ is a DE of the first order and degree 1.

$(y')^2 - 6xy' - 5y = 0$ is a DE of the first order and degree 2.

$(y'')^3 - (y')^5 x^2 - y^8 + 5x = 0$ is a DE of the second order and degree 3.

Differential equations – Types of a solution

DE can have three types of solutions:

- General solution
- Particular solution
- Singular solution

Differential equations – Types of a solution
Example 1

Find general solution of DE $y' =$ and particular solution for a condition $y(0) =$.

General solution:
We simply integrate DE: $y = x^2 +$

Particular solution for the initial condition: we substitute x = 0 and y = 2 into general solution:

$$2 = 0^2 +$$

Which yields C = 2. Thus, particular solution is $y = x^2 +$

Differential equations – Types of a solution
Example 2

Find general solution of DE $y' = \quad +$ and particular solution for a condition $y(1) = 2$.

General solution:
We integrate DE: $y = \quad + \quad +$

Particular solution for the initial condition: we substitute x = 1 and y = 2 into general solution:

$$2 = \quad + \quad +$$

Which yields C = - 2. Thus, particular solution is

$$y = \quad + \quad -$$

Differential equations – Types of a solution
Example 3

Find general solution of DE $y'' - \quad +$ and particular solution for a conditions $y(0) =$ and $y'(0) =$.

General solution: $y = \quad + \quad + \quad +$

Particular solution for the initial condition:

$y'(0) = \; = \; + \, +$ $\qquad y(0) = \; = \; + \, +$

Which yields $C_1 = 0$, $C_2 = 1$. Thus, particular solution is:

$$y = \quad + \quad +$$

Differential equations – Separation of variables

One of the most used method for solving DE is separation of variables. In this method x and y variables are separated on the different sides of an equation before integration takes place.

It can be used when DE is separable:

$$P(x) + Q(y)y' = 0 \quad \text{or} \quad P(x)dx + Q(y)dy = 0$$

Differential equations – Separation of variables
Example 1

Find a general solution of $yy' =$.

The equation is separable: $y\frac{dy}{dx} =$, so we separate both variables:

$$y\,dy =$$

And integrate: $\int \qquad \int$

Which yields:

$$\frac{y^2}{2} = \quad x^2 \quad +$$

Differential equations – Separation of variables
Example 2

Find a general solution of $y' + x - y =$.

The equation is separable, so we separate and integrate:

$$\frac{dy}{dx} + x - y =$$

$$\frac{dy}{dx} = - y$$

$$\frac{dy}{y} = - d$$

$$\int \frac{dy}{y} = \int (- 1)\, d +$$

$$y = \quad ^{2} - + \quad = \quad - \quad ^{2} -$$

Differential equations – Separation of variables
Example 3

Find a general solution of $\dfrac{y'}{x+3} - xy = 0$.

The equation is separable, so we separate and integrate:

$$\frac{dy}{(x+3)\,dx} - xy = 0$$

$$\frac{dy}{y} = x(x+3)\,dx$$

$$\int \frac{dy}{y} = \int (x^2+3x)\,dx + C$$

$$\ln|y| = \frac{x^3}{3} + \frac{3x^2}{2} + C$$

$$y = e^{\frac{x^3}{3}+\frac{3x^2}{2}+C} = Ae^{\frac{x^3}{3}+\frac{3x^2}{2}}$$

Differential equations – Homogenous differential equations

A DE of the form $y' =$ such that $f(tx,ty) =$ $)$ is called homogenous differential equation. It is solved via substitution: $y =$ and $y' = \quad +$.

Example: $x^2 y' = \quad -$ is homogenous, because:

$$f(tx,ty) = \quad - \quad = \quad - \quad = \quad -$$

Differential equations – Homogenous differential equations – Example 1

Find a general solution of a homogenous DE:

$$x^2 y' = \quad -$$

We start with the substitution $y =$:

$$u'x + \quad - \quad -$$

$$u'x - \quad - \quad -$$

$$\frac{du}{dx} x = \quad - \quad -$$

$$\frac{du}{u^2 \quad - \quad -} \quad dx$$

Differential equations – Homogenous differential equations – Example 1 – cont.

And at the end we integratate:

$$\int \frac{du}{\ } \quad \int \frac{dx}{\ }$$

$$\int \frac{du}{\ } \quad \int \frac{A}{\ } \quad \int \frac{B}{\ }$$

$$I_1 = \int \frac{du}{\ } \quad \frac{1}{\ }\int\frac{1}{\ } \quad \frac{1}{\ }\int\frac{1}{\ } \quad \ln u \quad u \quad +$$

which yields: $\frac{1}{3}\ln\left|\frac{u-}{u+}\right| \quad +$

Differential equations – Logistic equation and function

In economics, demographics and other disciplines appears a function called a logistic function.

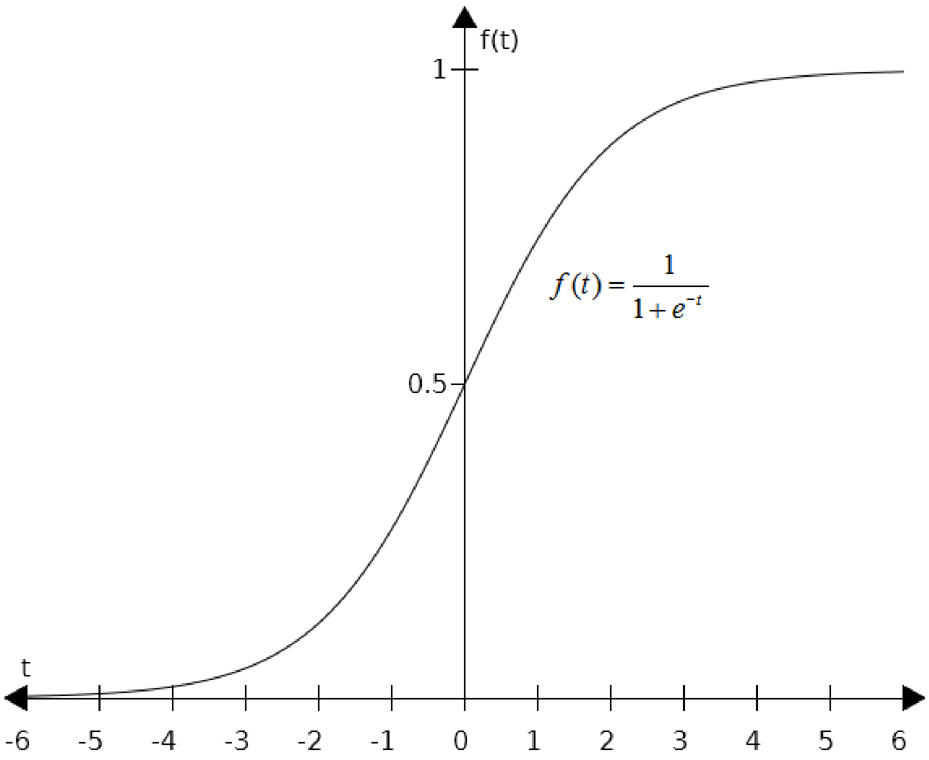
This function arises as a solution to the following logistic equation:

$$\frac{df}{dt} = \cdot -$$

For an initial condition $f(0) =$ the solution is:

$$f(t) =$$

Differential equations – Logistic equation and function



Differential equations

Linear differential equations of the first order

By a linear differential equations of the first order we mean an equation of the form:

$$y' + \quad =$$

Assume that q(x) = 0: $y' + \quad =$

This special equation is called homogenous, and is solved by separation of variables:

$$\frac{dy}{dx} = -$$

Differential equations

Linear differential equations of the first order – cont.

$$\frac{dy}{y} = -$$

$$\ln|y| = -\int$$

And finally we obtain:

$$y = \int$$

Differential equations

Linear differential equations of the first order – Example 1

Find the general solution: $y' + \quad = \quad .$

Solution:

We follow the procedure from the previous slide:

$$\frac{dy}{dx} = -$$

$$\frac{dy}{y} = -$$

$$\ln|y| = -\int$$

$$\ln|y| = - \quad + $$

$$y = \quad = \quad \cdot \quad =$$

Linear differential equations of the first order
Problems to solve

Find the general solution:

$$y' = \quad + \quad -$$

$$y' = \quad +$$

$$y'' = \quad +$$

$$xy' + 2y = 0$$

$$y' - \quad x \quad =$$

Thank you for your attention