Mathematics in Economics – lecture 2

1) Domain of a function of the form: (function with linear expression)

a) $f(x) = \frac{1}{2x-8}$

b) $f(x) = log(4x - 8)$

c) $f(x) = \sqrt{-3x + 4}$

2) Domain of a function of the form: (function with quadratic expression)

Firstly, we must decompose this expression into a product.

We can use the formula $a^2 - b^2 = (a - b)(a + b)$.

$9 - x^2 =$

$x^2 - 16 =$

We can factor out: $x^2 - 5x =$

$4x - x^2 =$

Solving the quadratic equation: $x^2 - 7x + 10 =$

$x^2 + 10x + 21 =$

Than we are going to solve quadratic inequation:

1) decomposition of expression into product

2) find zero points

3) find out what sign it takes in given interval (We choose number from interval and substitute it into the expression)

$x^2 - 36 \geq 0$

$7x - x^2 < 0$

$x^2 + 8x + 15 \leq 0$

Solve the domain of functions:

d) $f(x) = \frac{1}{x^2-25}$

e) $f(x) = log(6x - x^2)$

f) $f(x) = \sqrt{x^2 - 5x + 4}$

The derivative of a function (Derivatives of elementary functions)

$f(x)$	$f'(x)$
konstanta	0
x	1
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$\frac{1}{x}$
a^x	$a^x \cdot \ln a$
$\log_a x$	$\frac{1}{x \ln a}$

$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{tg} x$	$\frac{1}{\cos^2 x}$
$\operatorname{cotg} x$	$-\frac{1}{\sin^2 x}$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$
$\operatorname{arccotg} x$	$-\frac{1}{1+x^2}$

The basic rule is to put the exponent before the expression and then reduce the exponent by one.

$(2x^5)' =$

$(5x^4 - 3x^2 + 7x - 11)' =$

$(3x^{-4} - 2x^{-3} + 8x + 5)' =$

$(3sinx + 4cosx + 5lnx + 6e^x)' =$

The rules of differentiation

i) $\left[c \cdot f(x)\right]' = c \cdot f'(x)$

ii) $\left[f(x) \pm g(x)\right]' = f'(x) \pm g'(x)$

iii) $\left[f(x) \cdot g(x)\right]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

iv) $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}, \; g(x) \neq 0$

v) $\left[f(g(x))\right]' = f'(g(x)) \cdot g'(x)$

(multiplication, dividing, composite function)

$$y = \left(x^2 + 1\right) \cdot e^x$$

$$y = \left(x^2 + 4\right) \cdot \sin x$$

$$y = \frac{2x^2 - 3x + 1}{x}$$

$$y = \frac{x}{\ln x}$$

$$y = \ln\left(4x + 1\right)$$