Mathematics in Economics – lecture 9

Indefinite integral

Integration is a reverse procedure to differentiation.

Notation: $\int f(x)dx = F(x) + C$

Legend: $\int$ …. Integration sign – indefinite integral; $f(x)$ …. Integrated function;

$F(x)$ … antiderivative of $f(x)$; … C …. Integration constant

Indefinite integral is a linear operator:

$\int kf(x)dx = k\int f(x)dx$ $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

We compute integrals with the use of formulas above, and with the use of the table of elementary integrals:

Indefinite integral – elementary integrals

$f(x)$	$\int f(x)dx$
0	C
1	$x + C$
x^n	$\frac{x^{n+1}}{n+1} + C$
e^x	$e^x + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\frac{1}{ax+b}$	$\frac{1}{a}\ln\lvert ax+b\rvert + C$
a^x	$\frac{a^x}{\ln a} + C$

$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\frac{1}{\cos^2 x}$	$\text{tg}x + C$
$-\frac{1}{\sin^2 x}$	$\text{cotg}x + C$
$\frac{1}{1+x^2}$	$\text{arctg}x + C$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} arctg \frac{x}{a} + C$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x + C$
$-\frac{1}{\sqrt{1-x^2}}$	$\arccos x + C$
$\frac{1}{\sqrt{1\pm x^2}}$	$\ln\left\|x+\sqrt{1\pm x^2}\right\| + C$

Indefinite integral - examples

1) $\int\left(6x^2 + \sqrt{x}\right) dx =$

2) $\int(2x^2 + x - 15)dx =$

3) $\int(6x^3 + 7x^2 - 6x + 3)dx =$

4) $\int(3x + 1)(4x - 1)dx =$

5) $\int(2x + 3)^2 dx =$

6) $\int \frac{4x^3+3x^2-5x}{x^2} dx =$

7) $\boldsymbol{\int(x^3 + 2x^2 + 6x + 1)dx} =$

8) $\boldsymbol{\int \sqrt[3]{x}dx} = \boldsymbol{\int x^{\frac{1}{3}} dx} =$

9) $\boldsymbol{\int \left(2x + \frac{5}{x}\right) dx} =$

10) $\boldsymbol{\int(5 \sin x - 2 \cos x + 3^x)dx} =$

Indefinite integral – integration methods

For more complicated integration we use suitable integration methods:

- Substitutions
- Method per partes

All these methods will be demonstrated on examples.

I) Integration by a substitution

1) $\int (2x+1)^4 dx$

2) $\int e^{2x+3} dx$

3) $\int cos(5x-4)\, dx$

4) $\int \frac{ln\, x}{x} dx$

5) $\int \frac{5}{x\, ln^2 x} dx$

6) $\int \sqrt{4x+1} dx$

7) $\int x\sqrt{x^2-1} dx$

HOMEWORK

A] $\int (3x-2)^4 dx$

B] $\int sin(1-5x) dx$

C] $\int \frac{ln^2 x}{x} dx$

D] $\int cos^4 x\, sin\, x\, dx$

E] $\int \sqrt{2x+5} dx$

F] $\int x\sqrt{x^2+2} dx$

II) Integration by parts (per partes method)

Per partes method (integration by parts) is used for integration of a product of two functions.

Let $u(x)$ and $v(x)$ be two functions. Then, we obtain:

$$(u \cdot v)' = u'v + uv'$$

$$uv' = (uv)' - u'v$$

$$\int uv'\,dx = \int (uv)'\,dx - \int u'v\,dx$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

The last formula is "per partes" formula.

1) $\int x \cdot e^x dx$

$$\int x \cdot e^x dx = \begin{vmatrix} u = x, v' = e^x \\ u' = 1, v = e^x \end{vmatrix} = xe^x - \int 1 \cdot e^x dx = xe^x - e^x + C = (x-1)e^x + C$$

Note: a choice of u and v' is important. An incorrect choice leads to a growing difficulty of a problem.

2) $\int x \cdot ln\,x\,dx$

3) $\int x\,sin\,x\,dx$

HOMEWORK

A] $\int (2x+1).e^x dx$

B] $\int x^2.lnxdx$

C] $\int 3x.cosxdx$