

34.955	33.279	32.240
39.082	40.421	36.129
31.354	33.876	34.242
28.531	38.584	33.981
35.388	31.708	32.729
32.007	39.926	32.510
34.236	32.991	32.143
33.419	33.939	33.180
34.632	34.035	33.896
34.053	33.764	27.393
31.217	33.562	34.937
30.646	35.434	33.934
34.249	35.161	36.522
31.840	34.255	31.959
31.180	38.061	36.321
34.028	34.885	33.323
34.529	36.876	31.464
	34.585	35.055
		30.227

A group of 54 patients suffering from a disease. Each of the three subgroups was cured by a disease. The patients' bloods were tested to see the effect of the treatment. There are 17, 18, 19 patients in the first, second, and third groups. The values of  $\bar{x}_1, \bar{x}_2, \bar{x}_3$ , which are presented in the table, are the means of the blood tests of the first, second, and third groups.

- At the significance level of  $\alpha = 5\%$ , test the null hypothesis  $\mu_1 = \mu_2 = \mu_3$  that the means are equal.
- Formulate the alternative hypothesis that the means are not equal.
- Calculate:
  - the group sums and the group sample mean
  - the grand sum and the grand sample mean
- Calculate the quantity  $SS_B$  = the sum of squares between groups.
- Calculate the quantity  $SS_W$  = the sum of squares within groups.
- Calculate the Coefficient of Determination ( $R^2$ ). Is the fit "good"/"poor"?
- Determine the quantity  $DF_B$  = the degrees of freedom between groups.
- Determine the quantity  $DF_W$  = the degrees of freedom within groups.
- Calculate the quantity  $MS_B$  = the mean square between groups, i.e. the sample variance between the groups.
- Calculate the quantity  $MS_W$  = the mean square within groups, i.e. the sample variance within the groups.
- Calculate the statistic  $F = MS_B / MS_W = (SS_B / (k-1)) / (SS_W / (n-k))$ .
- Calculate the critical value  $F_{\alpha, k-1, n-k}$ .
- Calculate the  $p$ -value of the test.
- Do you reject or fail to reject the null hypothesis?

was divided into three subgroups.  
 distinct method. After the treatment,  
 effect of the respective treatment.  
 d, third group, respectively.  
 column A,B,C, are the results  
 up, respectively.  
 null hypothesis that all three treatments have the same effect.  
 all three treatments have the same effect.  
 effect of at least one treatment is different.

$IS$   
 $-$

es "between" = RegSS.

res "within" = RSS.

$\hat{\sigma}^2 = \text{RegSS} / \text{TSS}$ , where  $\text{TSS} = \text{RegSS} + \text{RSS}$ .

freedom "between".

freedom "within".

es "between",

$\sigma_s = SS_B / DF_B$ .

res "within",

$= SS_W / DF_W$ .

$(SS_W) / (DF_B / DF_W)$ .

esis?

3.8561	3.8993	3.7895
3.7656	4.2493	3.9607
3.7815	4.1771	3.8786
4.3176	4.0661	4.0504
3.9476	4.1042	3.7234
4.0355	4.0782	4.0493
3.9140	4.0601	3.8858
4.1162	3.9634	4.0049
4.1717	3.9194	4.0820
4.2424	4.1112	4.0099
4.0455	3.8839	3.7335
3.9302	4.2153	4.1394
3.7449	4.1208	3.9488
3.9567	3.8706	4.1098
4.1941	3.9350	4.1912
3.9546	3.8188	3.9844
3.7186	4.0522	4.0978
3.9927	4.3246	3.9921
4.0302	3.5822	3.9913
3.8388	3.8159	4.0615
3.7065	3.6214	3.9361
4.1795	4.2596	4.1131
3.7748	3.6856	4.3017
4.1145	4.1045	3.8698
3.8462	3.8758	3.9632
3.8228	4.2381	4.2873
4.1278	3.8984	4.1440
3.7701	3.6096	4.0555
3.9243	4.1651	3.8680
3.9039	3.7881	4.2965
4.1394	3.5936	3.9819
3.9595	4.1995	3.7590
3.7583	4.1139	3.8615
4.0545	3.8896	3.9272
	4.0341	4.1750
	3.9074	4.1189
	3.8565	
	3.8752	

We tested the mileage of three cars, and you are given the following data. You have 34 observations of the mileage of the first car. You have 38 observations of the mileage of the second car. You have 36 observations of the mileage of the third car. Assume for simplicity that the random error is normally distributed and has the same variance in each of the three samples (homoscedasticity).

- At the significance level of  $\alpha = 5\%$ , test the null hypothesis  $\mu_1 = \mu_2 = \mu_3$  that all three cars have the same mean mileage.
- Formulate the null hypothesis  $\mu_1 = \mu_2 = \mu_3$  that all three cars have the same mean mileage.
- Formulate the alternative hypothesis that the mileage of the three cars is not the same.
- Calculate:
  - the group sums and the group sample means  $\bar{y}_i$
  - the grand sum and the grand sample mean  $\bar{y}$
- Calculate the quantity  $SS_B$  = the sum of squares “between groups”
- Calculate the quantity  $SS_W$  = the sum of squares “within groups”
- Calculate the Coefficient of Determination ( $R^2 = \text{RegSS} / \text{TotalSS}$ ). Is the fit “good”/“poor”?
- Determine the quantity  $DF_B$  = the degrees of freedom “between groups”
- Determine the quantity  $DF_W$  = the degrees of freedom “within groups”
- Calculate the quantity  $MS_B$  = the mean squares “between groups” i.e. the the sample variance between the groups =  $SS_B / DF_B$
- Calculate the quantity  $MS_W$  = the mean squares “within groups” i.e. the the sample variance within the groups =  $SS_W / DF_W$
- Calculate the statistic  $F = MS_B / MS_W = (SS_B / DF_B) / (SS_W / DF_W)$
- Calculate the critical value  $F_{\alpha, DF_B, DF_W}$ .
- Calculate the  $p$ -value of the test.
- Do you reject or fail to reject the null hypothesis?

three samples of observations.

var.

distributed and  
homoskedasticity).

thesis that all three cars have the same results.  
cars have the same mileages.  
of at least one mileages is different.

between" = RegSS.

within" = RSS.

TSS / TSS, where TSS = RegSS+RSS).

"between".

"within".

between",

DF<sub>B</sub>.

n",

within".

F<sub>B</sub> / DF<sub>W</sub>).

178.8	208.3
181.4	211.9
178.1	210.4
179.8	212.5
179.4	212.0
180.8	208.5
184.4	213.2
180.3	210.6
178.3	211.3
180.4	211.3
178.2	216.5
178.6	211.6
183.8	209.0
177.4	209.3
178.6	208.3
179.0	209.7
181.8	211.0
180.8	212.8
181.6	207.9
179.6	212.4
179.3	212.8
181.7	208.7
181.2	209.1
181.1	212.4
184.4	211.7
176.4	210.6
178.0	208.2
174.0	211.9
183.9	211.4
180.4	212.6
182.7	
183.5	
181.8	
184.4	
180.3	
183.3	
177.7	
184.4	
180.8	
179.0	

You are given two samples of observations.  
 You have 40 observations of a random variable  
 (profit of company A observed during 40 consecutive  
 days) and 30 observations of random variable  
 (profit of company B observed during 30 consecutive days).  
 Assume for simplicity that both variables are normally distributed and  
 independent.

- Use the two sample  $t$ -test to test the null hypothesis that the (populatio  
 against the two-sided alternative hypothesis that they are different.  
 Calculate the  $p$ -value.

Considering the significance level  $\alpha = 1\%$ , do you reject or fail to reject the null hypothesis?

- Use the ANOVA  $F$ -test to test the null hypothesis that the (populations are equal)  
 against the two-sided alternative hypothesis that they are different.  
 Calculate the  $p$ -value.

Considering the significance level  $\alpha = 5\%$ , do you reject or fail to reject the null hypothesis?

and have the same variance.  
(variance) means  $\mu_1$  and  $\mu_2$  are the same,

reject the null hypothesis?  
(variance) means  $\mu_1$  and  $\mu_2$  are the same,

reject the null hypothesis?

61.29	55.25
54.98	59.45
55.94	63.79
56.43	59.01
55.61	61.03
54.49	62.74
59.99	59.13
58.47	63.57
60.32	65.25
58.67	62.90
60.93	64.04
57.66	58.20
53.87	57.64
55.45	60.51
66.02	60.17
69.34	58.30
59.44	54.09
61.95	55.43
64.73	63.46
60.13	58.06
59.56	62.10
63.28	60.36
63.71	54.77
59.63	56.66
56.14	59.04
54.62	63.71
61.14	63.31
60.84	57.31
62.28	59.99
57.91	57.02
	60.03
	57.96
	57.40
	61.74
	62.90
	63.06
	63.13
	59.21
	62.39
	59.03

You are given two samples of observations.  
 You have 30 observations of a random variable (profit of company A observed during 30 consecutive days) and 40 observations of random variable (profit of company B observed during 40 consecutive days).  
 Assume for simplicity that both variables are normally distributed and independent.

- Use the two sample  $t$ -test to test the null hypothesis that the (population) means are equal against the two-sided alternative hypothesis that they are different. Calculate the  $p$ -value. Considering the significance level  $\alpha = 5\%$ , do you reject or fail to reject the null hypothesis?
- Use the ANOVA  $F$ -test to test the null hypothesis that the (population) means are equal against the two-sided alternative hypothesis that they are different. Calculate the  $p$ -value. Considering the significance level  $\alpha = 1\%$ , do you reject or fail to reject the null hypothesis?

and have the same variance.  
(variance) means  $\mu_1$  and  $\mu_2$  are the same,

reject the null hypothesis?  
(variance) means  $\mu_1$  and  $\mu_2$  are the same,

reject the null hypothesis?