APcol systems and Turtle Graphics

Workshop on Natural Computing , ITAT 2021

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Extended APcol systems

- Concept
- Rules
- Computation

Space-filling curves

- Hilbert curve
- Moore curve
- Peano curve

Conclusion



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Extended APcol systems

Introduction



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APcol systems also called Automaton-like P colonies - they belong to a family of models called membrane systems or P systems.

Membrane systems

Introduced by Gheorghe Păun in 2000[1]. The system consists of a structure of **membranes**, each containing **multiset of objects** and equipped with a **set of rules or programs** by which the objects are transformed and transported.

[1] Gheorghe Păun. "Computing with Membranes". In: *Journal of Computer and System Sciences* 61.1 (2000), pp. 108–143.

Introduction



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APcol systems also called Automaton-like P colonies - they belong to a family of models called membrane systems or P systems.

P colonies

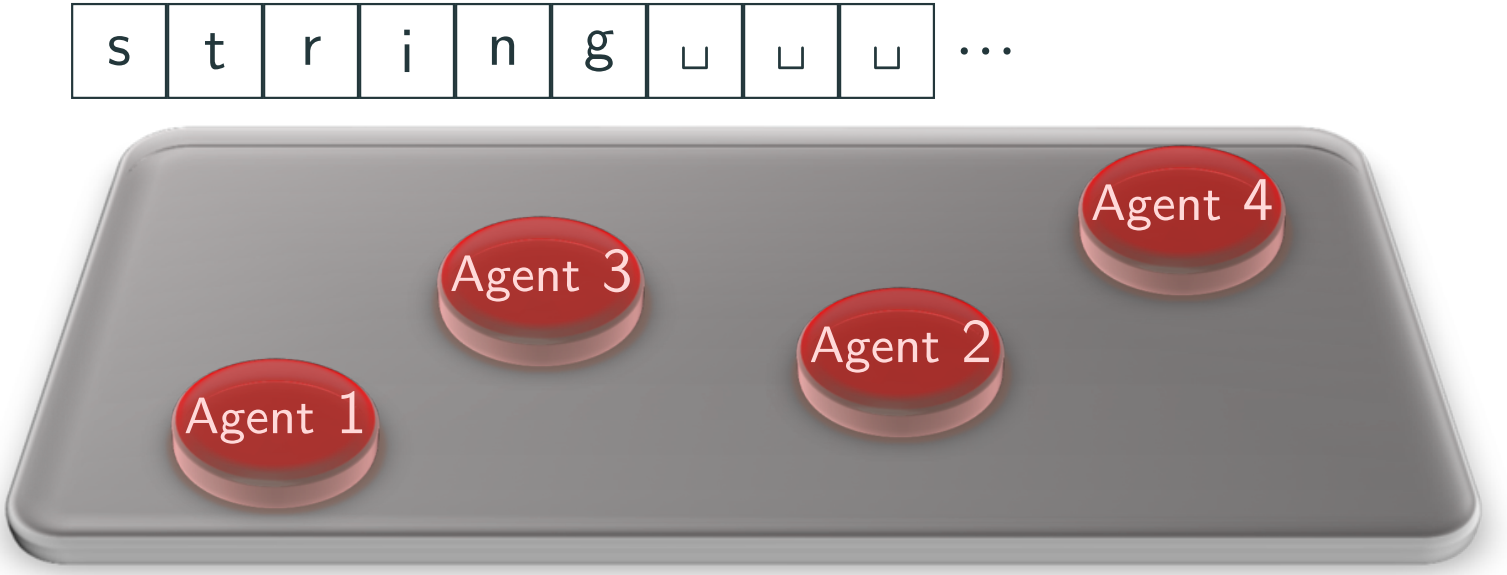
Introduced by Gheorghe Păun, Alica Kelemenová and Jozef Kelemen in 2000[2]. The system consists of **agents** - one membrane cells, each containing **multiset of objects** of given cardinality and they are equipped with a **set of programs**.

[2] J. Kelemen, A. Kelemenová, and Gh. Păun. "Preview of P colonies: A biochemically inspired computing model". In: *Workshop and Tutorial Proceedings. Ninth International Conference on the Simulation and Synthesis of Living Systems (Alife IX)*. Boston, Mass, 2004, pp. 82–86.

Introduction

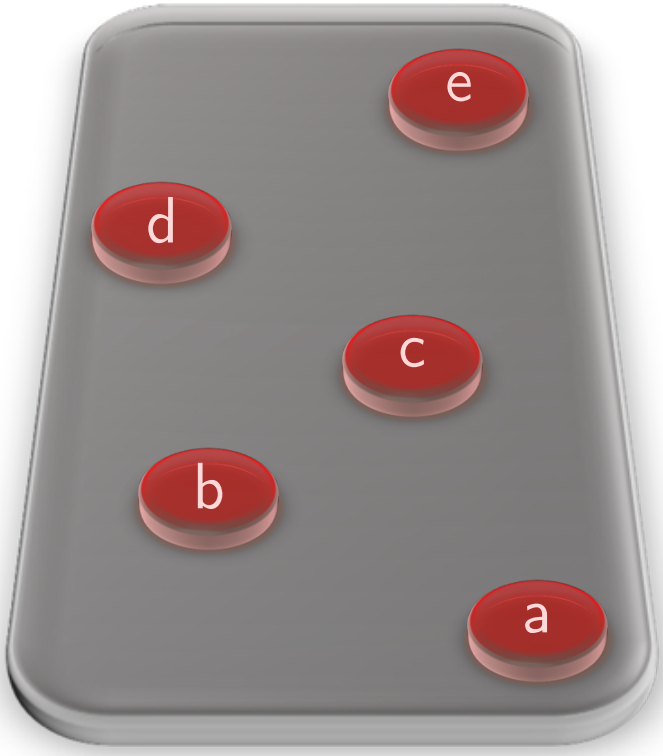
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Automaton-like P colonies - an environment in a form of string



Agent

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- collection of objects
- embedded in a membrane
- with set of programs

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Rules

- Rewriting rule $a \longrightarrow b$

  Every agent can change its content by evolving it.

  Agent:abc Rule:$a \longrightarrow b$ Agent:bbc

  Env.: bbd Env.: bbd

- communication rule $c \longleftrightarrow d$

  Every agent can change its state and content of environment by communication.

  Agent:abc Rule:$c \longleftrightarrow d$ Agent:abd

  Env.: bbd Env.: bbc

Introduction

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Rules

- object occurrence conditional rule $o|r$ or $\neg o|r$, where r is rewriting or communication rule

  Agent can apply rule r only if it is applicable and object o is present (or not present) in the environmental string.

Agent:abc	Rule:$o\|\ c \longleftrightarrow d$	Agent:abc
Env.: bbd		Env.: bbd
Agent:abc	Rule:$o\|\ c \longleftrightarrow d$	Agent:abd
Env.: $bbdo$		Env.: $bbco$
Agent:abc	Rule:$\neg o\|\ c \longleftrightarrow d$	Agent:abd
Env.: bbd		Env.: bbc

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Agent creation

If an agent contains a special object @, the agent makes a copy of itself.

Agent: $@bc$ Rules: $b \to d; c \to d;\ @ \to d$ Agent: ddd
Env.: bbd Agent: ddd
Env.: bbd

Agent: $@bc$ Rules: $@ \to d;\ b \to d;\ c \to d$ Agent: dbc
Env.: bbd Agent: ddd
Env.: bbd

Capacity of cell

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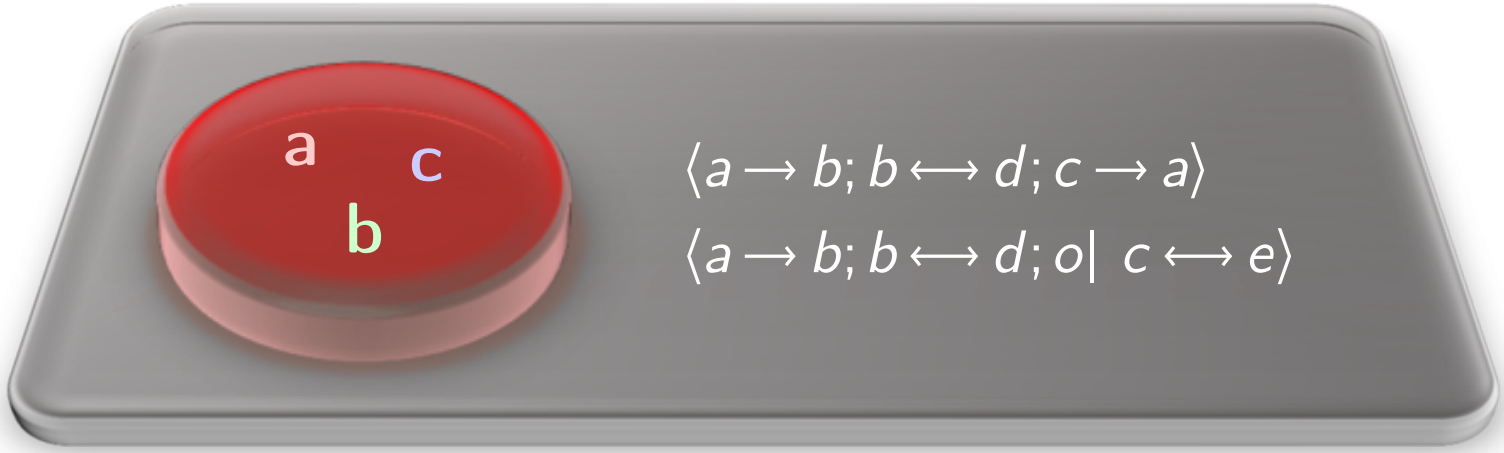
- The number of objects inside the cell is constant during a computation.
- All cells have the same number of objects inside them
- This number is called **capacity** of P Colony
- A capacity sets the number of rules in program



Capacity of cell

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Space-filling curves and Turtle graphics

Turtle Graphics



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Turtle is defined as a triplet (x, y, α), where

- (x, y) - the Cartesian coordinates
- the angle α - the direction in which the turtle is facing

Given the step size d and the angle increment δ, the turtle can respond to commands represented by the following symbols:

F	Move forward a step of length d.
f	Move forward a step of length d without drawing a line.
$+$	Turn left by angle δ. The positive orientation of angles is counter-clockwise.
$-$	Turn right by angle δ.

Space-filling curves



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In this contribution we focus on fractal-like space-filling curves of three types - Hilbert, Moore and Peano curve.

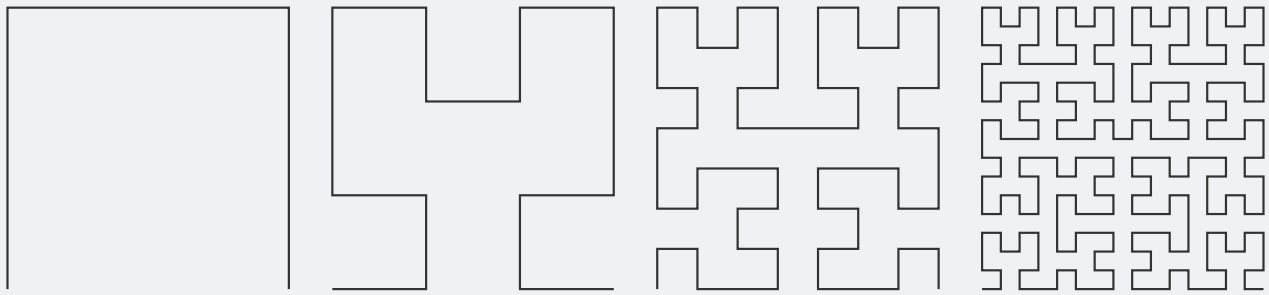
We showed how the extended APcol system generating command string for turtle is constructed.

Space-filling curves

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Hilbert curve

In 1891 Hilbert introduced the Hilbert space-filling curve.



The approximations can be described by Hilbert words - chain-code words.

- $H_1 = urd$
- $H_{n+1} = \gamma(H_n)uH_nrH_nd\varphi(H_n)$

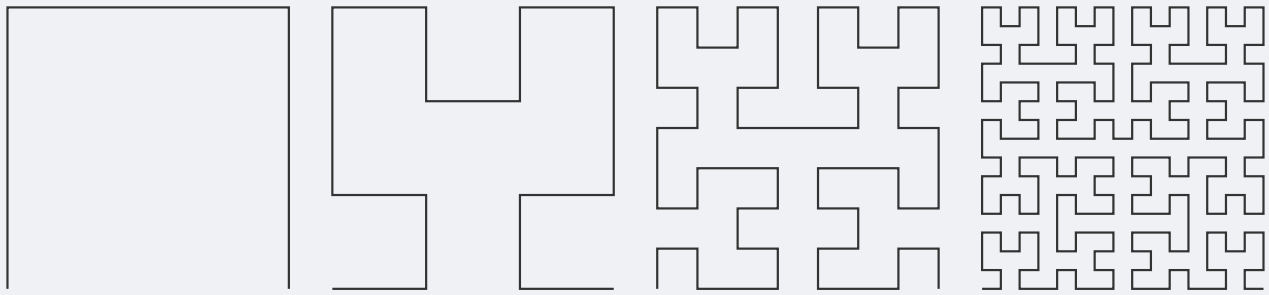
γ – symmetry w.r.t. O_y and rotation right 90 degrees;

φ – symmetry w.r.t. O_y and rotation left 90 degrees.

Space-filling curves

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Hilbert curve



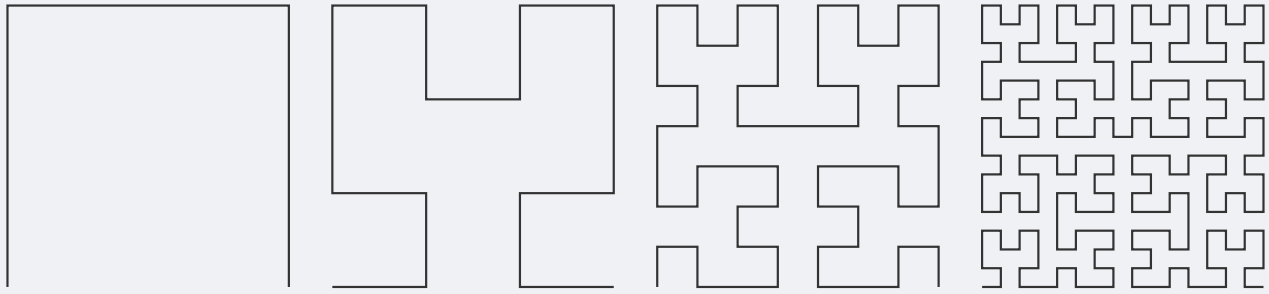
Parallel grammar with non-terminal alphabet $\{N, L, R, U\}$, terminals from $\{u, d, l, r\}$ and having two groups of CF rules:

$$
\begin{array}{rcl@{\qquad\qquad}rcl}
N & \to & LuNrNdR & N & \to & \varepsilon \\
L & \to & NrLuLlU & L & \to & \varepsilon \\
R & \to & UlRdRrN & R & \to & \varepsilon \\
U & \to & RdUlUuL & U & \to & \varepsilon
\end{array}
$$

Space-filling curves

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Hilbert curve



Parallel grammar with non-terminal alphabet $\{N, L, R, U\}$, terminals from $\{u, d, l, r\}$ and having two groups of CF rules:

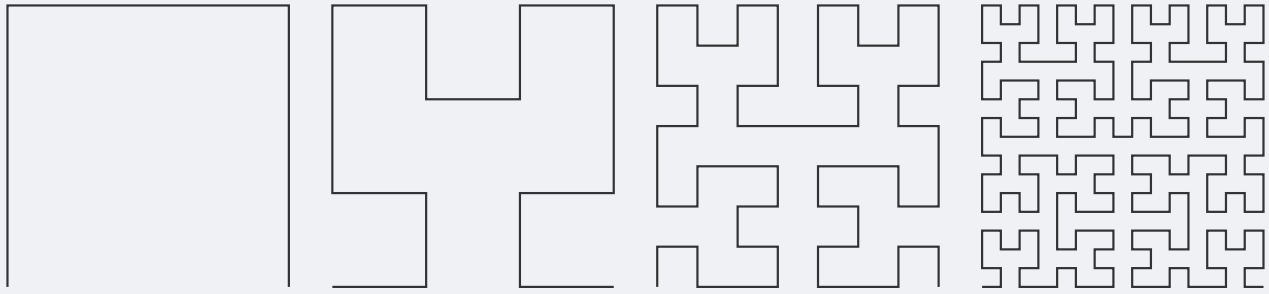
$$\begin{array}{rcl} N & \to & LuNrNdR \\ L & \to & NrLuLlU \\ R & \to & UlRdRrN \\ U & \to & RdUlUuL \end{array} \qquad \begin{array}{rcl} N & \to & \varepsilon \\ L & \to & \varepsilon \\ R & \to & \varepsilon \\ U & \to & \varepsilon \end{array}$$

$N \Rightarrow LuNrNdR \Rightarrow urd$

Space-filling curves

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Hilbert curve



Parallel grammar with non-terminal alphabet $\{N, L, R, U\}$, terminals from $\{u, d, l, r\}$ and having two groups of CF rules:

$$\begin{array}{rcl} N & \to & LuNrNdR \\ L & \to & NrLuLlU \\ R & \to & UlRdRrN \\ U & \to & RdUlUuL \end{array} \qquad \begin{array}{rcl} N & \to & \varepsilon \\ L & \to & \varepsilon \\ R & \to & \varepsilon \\ U & \to & \varepsilon \end{array}$$

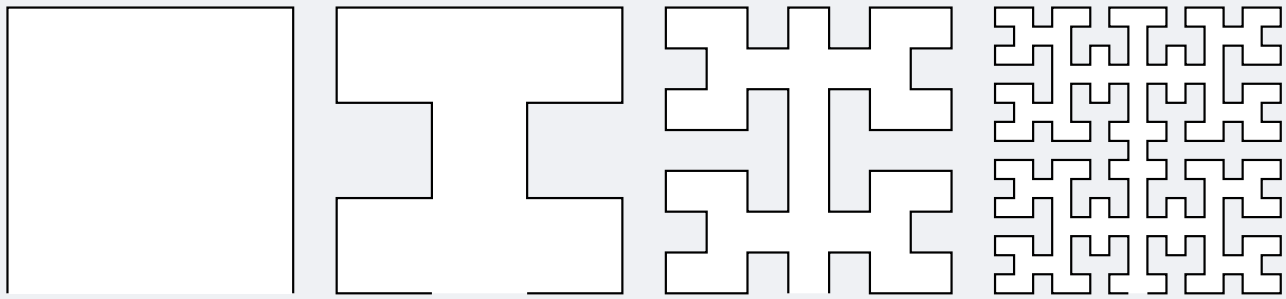
$N \Rightarrow LuNrNdR \Rightarrow NrLuLlUuLuNrNdRrLuNrNdRdUlRdRrN \Rightarrow$
$ruluurdrurddldr$

Space-filling curves

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Moore curve

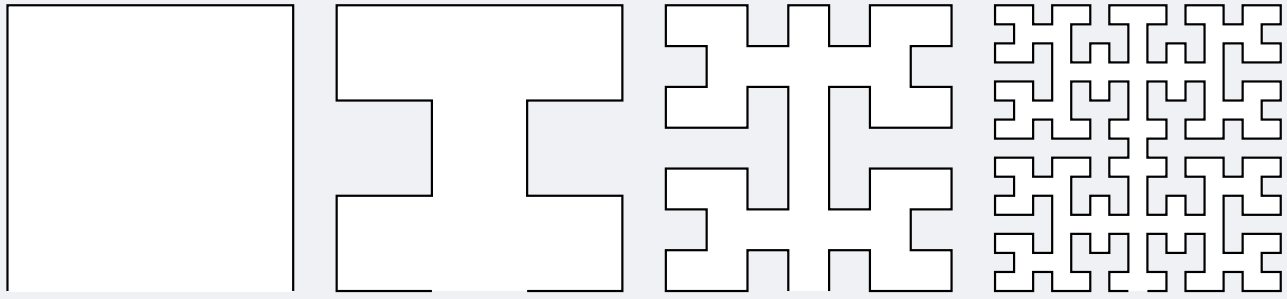
The Moore curve is a variant of the Hilbert space-filling curve.



Space-filling curves

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Moore curve



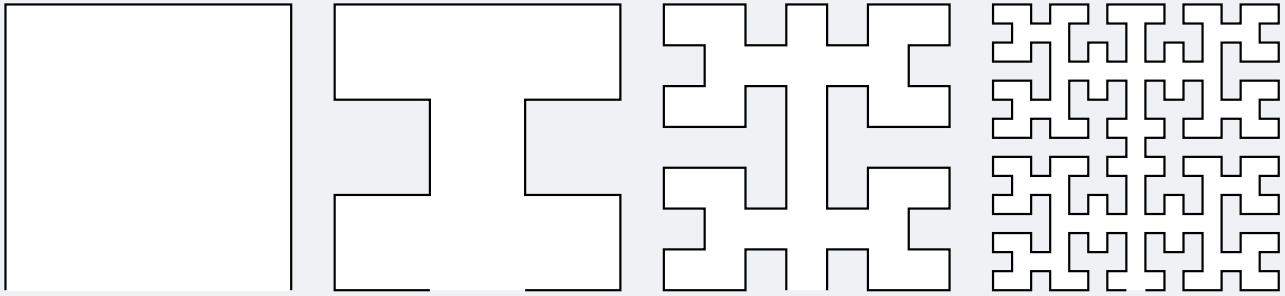
Parallel grammar with non-terminal alphabet $\{A, B, C, E, F\}$, terminals from $\{u, d, l, r\}$ and having two groups of CF rules:

$$
\begin{array}{lcl}
A & \to & BuBrGdG \\
B & \to & ClBuBrE \\
C & \to & BuClCdG \\
E & \to & GdErEuB \\
G & \to & ErGdGlC
\end{array}
\qquad
\begin{array}{lcl}
A & \to & \varepsilon \\
B & \to & \varepsilon \\
C & \to & \varepsilon \\
E & \to & \varepsilon \\
G & \to & \varepsilon
\end{array}
$$

Space-filling curves

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Moore curve



Parallel grammar with non-terminal alphabet $\{A, B, C, E, F\}$, terminals from $\{u, d, l, r\}$ and having two groups of CF rules:

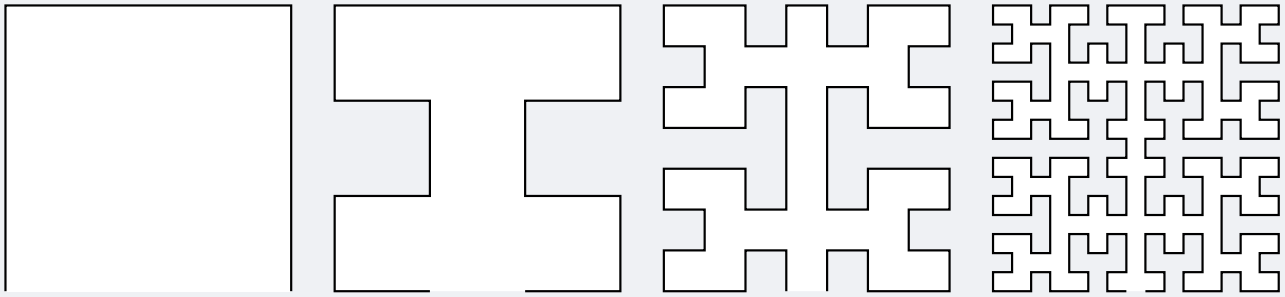
A	$\rightarrow$	$BuBrGdG$	A	$\rightarrow$	ε
B	$\rightarrow$	$ClBuBrE$	B	$\rightarrow$	ε
C	$\rightarrow$	$BuClCdG$	C	$\rightarrow$	ε
E	$\rightarrow$	$GdErEuB$	E	$\rightarrow$	ε
G	$\rightarrow$	$ErGdGlC$	G	$\rightarrow$	ε

$A \Rightarrow BuBrFdF \Rightarrow urd$

Space-filling curves

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Moore curve



Parallel grammar with non-terminal alphabet $\{A, B, C, E, F\}$, terminals from $\{u, d, l, r\}$ and having two groups of CF rules:

A	$\rightarrow$	$BuBrGdG$	A	$\rightarrow$	ε
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C	$\rightarrow$	$BuClCdG$	C	$\rightarrow$	ε
E	$\rightarrow$	$GdErEuB$	E	$\rightarrow$	ε
G	$\rightarrow$	$ErGdGlC$	G	$\rightarrow$	ε

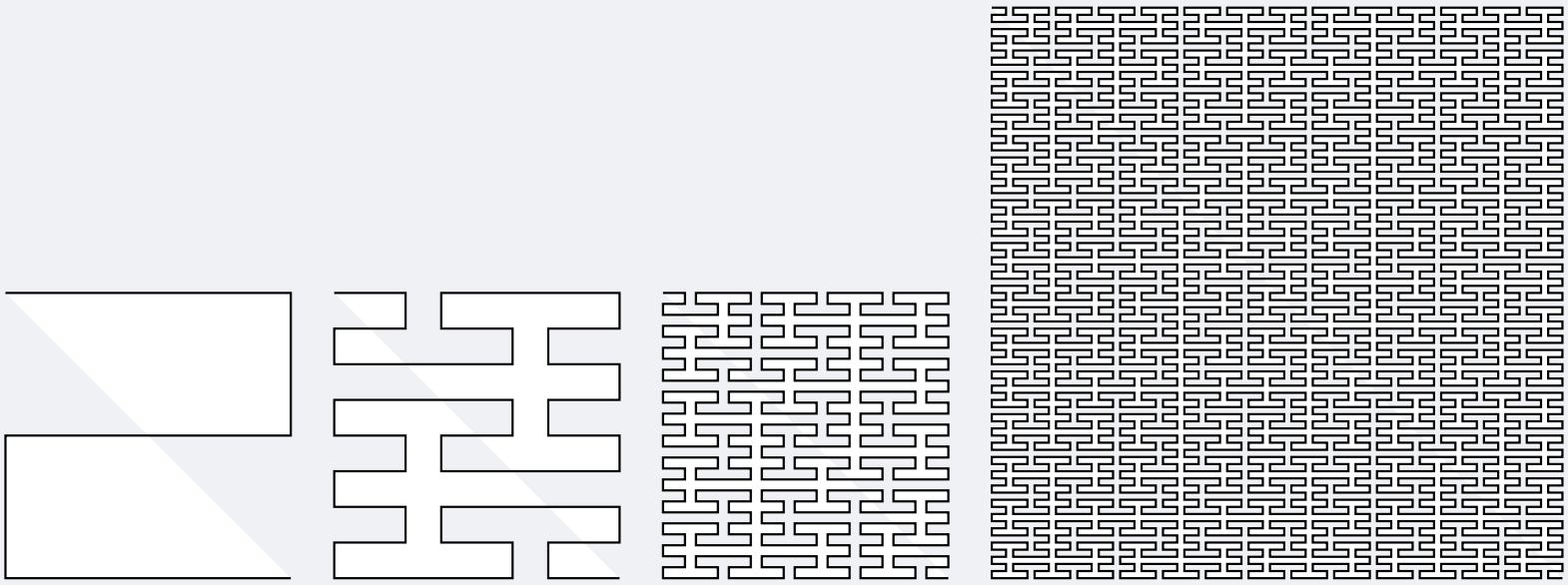
$A \Rightarrow BuBrGdG \Rightarrow ClBuBrEuClBuBrErErGdGlCdErGdGlC \Rightarrow$
$lurulurrrdldrd$

Space-filling curves

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Peano curve

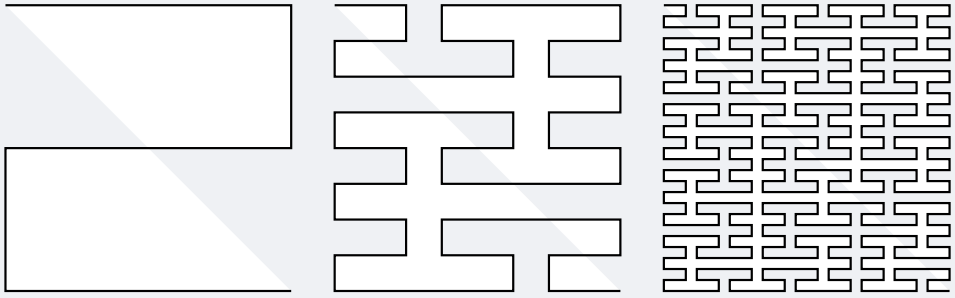
Peano defined this space-filling curve in 1890.



Space-filling curves

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Peano curve



Parallel grammar with non-terminal alphabet $\{N, L, R, D\}$, terminals from $\{u, d, l, r\}$ and having two groups of CF rules:

$$
\begin{array}{rcl@{\qquad\qquad}rcl}
N & \to & NuLuNrRdDdRrNuLuN & N & \to & \varepsilon \\
L & \to & LuNuLlDdRdDlLuNuL & L & \to & \varepsilon \\
R & \to & RdDdRrNuLuNrRdDdR & R & \to & \varepsilon \\
D & \to & DdRdDlLuNuLlDdRdD & D & \to & \varepsilon
\end{array}
$$



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eAPcol system and Hilbert Curve

General

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- eApcol system with capacity 3 has two different agents A_1 and A_2
- environmental string : $a^n\#$, n-th approximation of HC
- 4 phases of generation of command string: initialization, derivation, agent creation, coding to command string for turtle

Initialization

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Example

Configuration			Applicable programs	
Env.	A_1	A_2	A_1	A_2
$a\#$	eee	eee	$<\#\|e \rightarrow e; e \rightarrow e; e \rightarrow b>$	$<e \rightarrow \#_1, e \rightarrow N; e \rightarrow e>$
$a\#$	eeb	$\#_1 Ne$	$<e \rightarrow e; e \rightarrow e; b \rightarrow c>$	$<\#_1 \leftrightarrow \#; a\|N \leftrightarrow e; e \rightarrow e>$
$a\#_1 N$	eec	$\#ee$	$-$	$<\# \rightarrow \#_X, e \rightarrow e; e \rightarrow e>$

Derivation

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Example

Configuration			Applicable programs	
Env.	A_1	A_2	A_1	A_2
$a\#_1 N$	eec	$\#ee$	–	$B3_{(X=N)}$
$a\#_1 N$	eec	$\#_N ee$	–	$B4$
$a\#_1\#_N$	eec	$N\#_{N_1} ee$	$A3$	$B6$
$a\#_1\#_N$	$w_1 ee$	$\#_{N_1} LN_1$	$A4$	–
$a\#_1\#_N$	$w_2 ee$	$\#_{N_1} LN_1$	$A5$	–
$a\#_1\#_N$	$w_3 ee$	$\#_{N_1} LN_1$	$A6$	–
$a\#_1\#_N$	$w_4 ee$	$\#_{N_1} LN_1$	$A7$	–
$a\#_1\#_N$	$0ge$	$\#_{N_1} LN_1$	$A8$	–

Derivation

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Example

Configuration			Applicable programs	
Env.	A_1	A_2	A_1	A_2
$g\#_1\#_N$	①ae	$\#_{N_1}LN_1$	$A11$	$B7_{(x_1=L)}$
$\#_1L\#_{N_1}$	②eg	$\#_N eN_1$	$A12$	$B8_{(x_2=u)}$
$\#_1L\#_{N_1}$	e③e	$\#_{N_2}uN_2$	$A13_{(i=3)}$	$B9$
$\#_1Lu\#_{N_2}$	e④e	$\#_{N_1}eN_2$	$A13_{(i=4)}$	$B10_{(i=3;\ x_3=N)}$
$\#_1Lu\#_{N_2}$	e⑤e	$\#_{N_3}NN_3$	$A13_{(i=5)}$	$B11_{(i=3)}$
$\#_1LuN\#_{N_3}$	e⑥e	$\#_{N_2}eN_3$	$A13_{(i=6)}$	$B10_{(i=4;\ x_4=r)}$
$\#_1LuN\#_{N_3}$	e⑦e	$\#_{N_4}rN_4$	$A13_{(i=7)}$	$B11_{(i=4)}$
$\#_1LuNr\#_{N_4}$	e⑧e	$\#_{N_3}eN_4$	$A13_{(i=8)}$	$B10_{(i=5;\ x_5=N)}$

Derivation



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Example

Configuration			Applicable programs	
Env.	A_1	A_2	A_1	A_2
$\#_1 LuNr\#_{N_4}$	$e⑨e$	$\#_{N_5} NN_5$	$A13(i{=}9)$	$B11(i{=}5)$
$\#_1 LuNrN\#_{N_5}$	$e⑩e$	$\#_{N_4} eN_5$	$A13(i{=}10)$	$B10(i{=}6;\ x_6{=}d)$
$\#_1 LuNrN\#_{N_5}$	$e⑪e$	$\#_{N_6} dN_6$	$A13(i{=}11)$	$B11(i{=}6)$
$\#_1 LuNrNd\#_{N_6}$	$e⑫e$	$\#_{N_5} eN_6$	$A13(i{=}12)$	$B10(i{=}7;\ x_7{=}R)$
$\#_1 LuNrNd\#_{N_6}$	$e⑬e$	$\#_{N_7} RN_7$	$A13(i{=}13)$	$B12$
$\#_1 LuNrNdR$	$e⑭e$	$\#_{N_6} eN_7$	$A13(i{=}14)$	$B13$
$\#_1 LuNrNdR$	$e⑮e$	$@Fe$	$A14$	$B14$

Derivation

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Example

Configuration			Applicable programs	
Env.	A_1	A_2	A_1	A_2
text				

Agent Creation



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Example

	Configuration				
Env.	A_1	A_2	A_2	A_2	A_2
$\#_1 LuNrNdR$	$e\textcircled{15}e$	$@Fe$	–	–	–
$\#_1 LuNrNdR$	eec	$S@e$	$S@e$	–	–
$\#_1 LuNrNdR$	eec	$ee\#$	$ee\#$	$ee\#$	$ee\#$

End of derivation

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Example

	Configuration				
Env.	A_1	A_2	A_2	A_2	A_2
$\#_1 LuNrNdR$	eec	$ee\#$	$ee\#$	$ee\#$	$ee\#$
$\#_1 LuNrNdR$	eec	$ee\#_L$	$ee\#_N$	$ee\#_N$	$ee\#_R$
$\#_1\#_L u\#_N r\#_N d\#_R$	eec	$L\#_{L_1} e$	$N\#_{N_1} e$	$N\#_{N_1} e$	$R\#_{R_1} e$
$\#_1\#_L u\#_N r\#_N d\#_R$	eew_1	$N\#_{L_1} L_1$	$L\#_{N_1} N_1$	$L\#_{N_1} N_1$	$U\#_{R_1} R_1$
$\#_1\#_L u\#_N r\#_N d\#_R$	eew_2	$N\#_{L_1} L_1$	$L\#_{N_1} N_1$	$L\#_{N_1} N_1$	$U\#_{R_1} R_1$
$\#_1\#_L u\#_N r\#_N d\#_R$	eew_3	$N\#_{L_1} L_1$	$L\#_{N_1} N_1$	$L\#_{N_1} N_1$	$U\#_{R_1} R_1$
$\#_1\#_L u\#_N r\#_N d\#_R$	eew_4	$N\#_{L_1} L_1$	$L\#_{N_1} N_1$	$L\#_{N_1} N_1$	$U\#_{R_1} R_1$
$\#_1\#_L u\#_N r\#_N d\#_R$	$ef0'$	$N\#_{L_1} L_1$	$L\#_{N_1} N_1$	$L\#_{N_1} N_1$	$U\#_{R_1} R_1$
$f\#_L u\#_N r\#_N d\#_R$	$e\#_1 0'$	$N\#_{L_1} L_1$	$L\#_{N_1} N_1$	$L\#_{N_1} N_1$	$U\#_{R_1} R_1$
$f\#_L u\#_N r\#_N d\#_R$	$\boxed{u}\#_2 e$	feL_1	feN_1	feN_1	feR_1
$furd$	$\boxed{u}\#_2 e$	$ff\#_L$	$ff\#_N$	$ff\#_N$	$ff\#_R$

Coding

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Example

$A15$ $< 0' \rightarrow \boxed{u}; \#_1 \rightarrow \#_2; e \rightarrow e >;$

to move in the same direction as turtle is facing

$x \in \{u, d, r, l\}$

$A16$ $< \boxed{x} \rightarrow \boxed{x}; \#_2 \leftrightarrow f; e \leftrightarrow x >;$

$A17$ $< \boxed{x} \rightarrow \boxed{x}; f \rightarrow f; x \rightarrow F >;$

$A18$ $< \boxed{x} \rightarrow \boxed{x}; F \leftrightarrow \#_2; f \leftrightarrow e >;$

Coding

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Example

to rotate and move

$cc(u) = r; cc(r) = d; cc(d) = l; cc(l) = u; p = cc(x)$

$A19 \quad < \boxed{x} \rightarrow \boxed{x}; \#_2 \leftrightarrow f; e \leftrightarrow p >;$

$A20 \quad < \boxed{x} \rightarrow \boxed{p}; f \rightarrow f'; p \rightarrow - >;$

$A21 \quad < \boxed{p} \rightarrow \boxed{p}; - \leftrightarrow \#_2; f' \leftrightarrow e >;$

$A22 \quad < \boxed{p} \rightarrow \boxed{p}; \#_2 \leftrightarrow f'; e \rightarrow F >;$

$A23 \quad < \boxed{p} \rightarrow \boxed{x}; f' \rightarrow f; F \rightarrow F >;$

$cw(u) = l; cw(l) = d; cw(d) = r; cw(r) = u; q = cw(x)$

$A24 \quad < \boxed{x} \rightarrow \boxed{x}; \#_2 \leftrightarrow f; e \leftrightarrow q >;$

$A25 \quad < \boxed{x} \rightarrow \boxed{q}; f \rightarrow f'; q \rightarrow + >;$

$A26 \quad < \boxed{q} \rightarrow \boxed{q}; + \leftrightarrow \#_2; f' \leftrightarrow e >;$

$A27 \quad < \boxed{q} \rightarrow \boxed{q}; \#_2 \leftrightarrow f'; e \rightarrow F >;$

$A28 \quad < \boxed{q} \rightarrow \boxed{x}; f' \rightarrow f; F \rightarrow F >;$

Coding

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Example

to rotate twice and move

$dc(u) = d; dc(l) = r; dc(d) = u; dc(r) = l; y = dc(x)$

$A29 \quad < \boxed{x} \rightarrow \boxed{x}; \#_2 \leftrightarrow f; e \leftrightarrow y >;$

$A30 \quad < \boxed{x} \rightarrow \boxed{y}; f \rightarrow f''; y \rightarrow + >;$

$A31 \quad < \boxed{y} \rightarrow \boxed{y}; + \leftrightarrow \#_2; f'' \leftrightarrow e >;$

$A32 \quad < \boxed{y} \rightarrow \boxed{y}; \#_2 \leftrightarrow f''; e \rightarrow + >;$

$A33 \quad < \boxed{y} \rightarrow \boxed{q}; f'' \rightarrow f'; + \rightarrow + >;$

Moore and Peano curve



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Moore curve

It differs in the number of CF rules and in left-hand side of the rules not in the number of non-terminals on the left-hand side of the rules.

Peano curve

The number of non-terminals on left-hand side of the rule is greather than four, agent creation phase has more steps (the agents have to reproduce themselves more times.)

Conclucion



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Extended APcol systems

Space-filling curves and Turtle graphics

eAPcol system and Hilbert Curve

Moore and Peano curves



Questions? Comments?



Thank you for your attention!