On shapes of van Stockum dust clouds

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History

K. Lanczos (1924) – found exact stationary cylindrically symmetric dust solution (“cylindrical world”).

W.J. van Stockum (1937) – found general stationary axisymmetric dust depending on a solution of the cylindric Laplace equation – matched the Lanczos (1924) dust to one of the Lewis (1932) vacua to obtain a stationary cylindrical dust cloud.

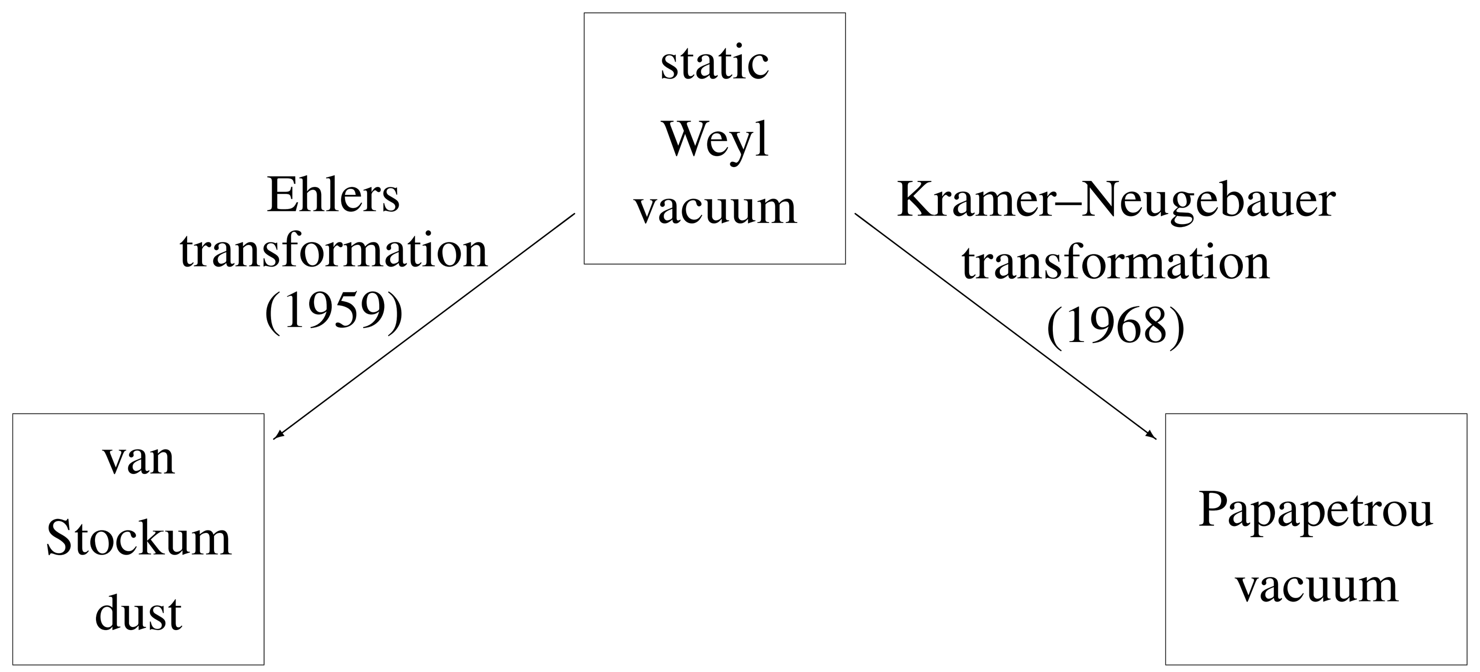
W.B. Bonnor (1977) – studied the simplest stationary axisymmetric asymptotically flat dust solution.

W.B. Bonnor (1982) – proved that the 1977 dust gathers around a singularity the negative mass of which exactly balances the positive mass of the dust. Asked for a matching vacuum.

J. Zsigrai (2003) – matched the Lukács–Newman–Sparling–Winicour (1983) dust to the Newman–Unti–Tamburino vacuum.

M.M. (2023) – matched the general van Stockum solution to a Papapetrou vacuum and the converse (this poster).

Main result



Theorem (M.M. 2023). *Given a static Weyl vacuum (henceforth the seed), then its Ehlers dust and Kramer–Neugebauer vacuum counterparts match along the equipotential surfaces of the seed.*

Two matching metrics produce a composite metric of class C^1.

Examples

A general Weyl’s solution depends on an axisymmetric solution of the Laplace equation. These admit a very rich topology of level sets = dust boundaries (Enciso and Peralta-Salas, 2013).

1. Bonnor–Bonnor. Bonnor dust (1977) matches to Bonnor vacuum (2005), unbeknownst to the author himself. The static Weyl seed is the gravitational dipole.

2. Bach–Weyl. The static Bach–Weyl ring (1922) generates both elliptic and toroidal dust shapes as well as elliptic clouds possessing a toroidal hole.

3. Appell–Gleiser–Pullin. The static Weyl solution due to Gleiser and Pullin (1989), based on Appell’s (1887) harmonic function, gives rise to even more complex shapes.

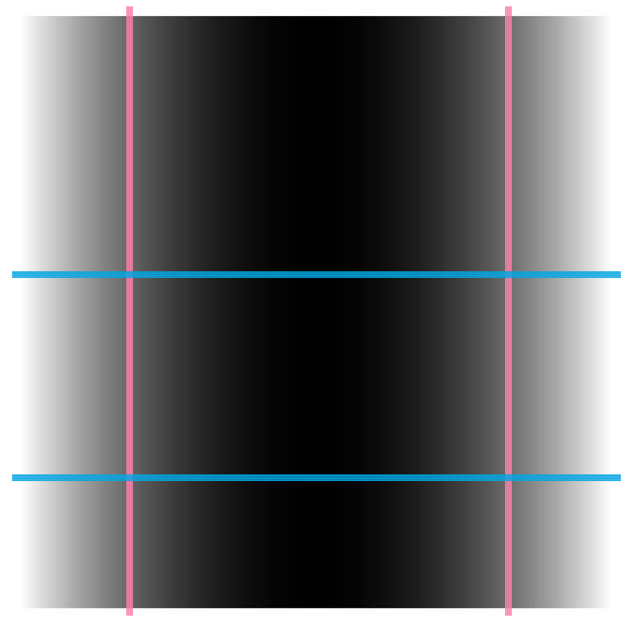
See columns 1–3 in the picture on the right.

Generic solutions are axisymmetric (admit a regular axis).

4. The **Chazy–Curzon** static “particle” generates a matching pair the dust part of which is the Halilsoy “spinning particle” (1992). This metric has no regular axis, however.

Incompleteness

It turns out that **Lanczos’s dust** (1924) admits at least two sets of boundaries



Van Stockum (1937)

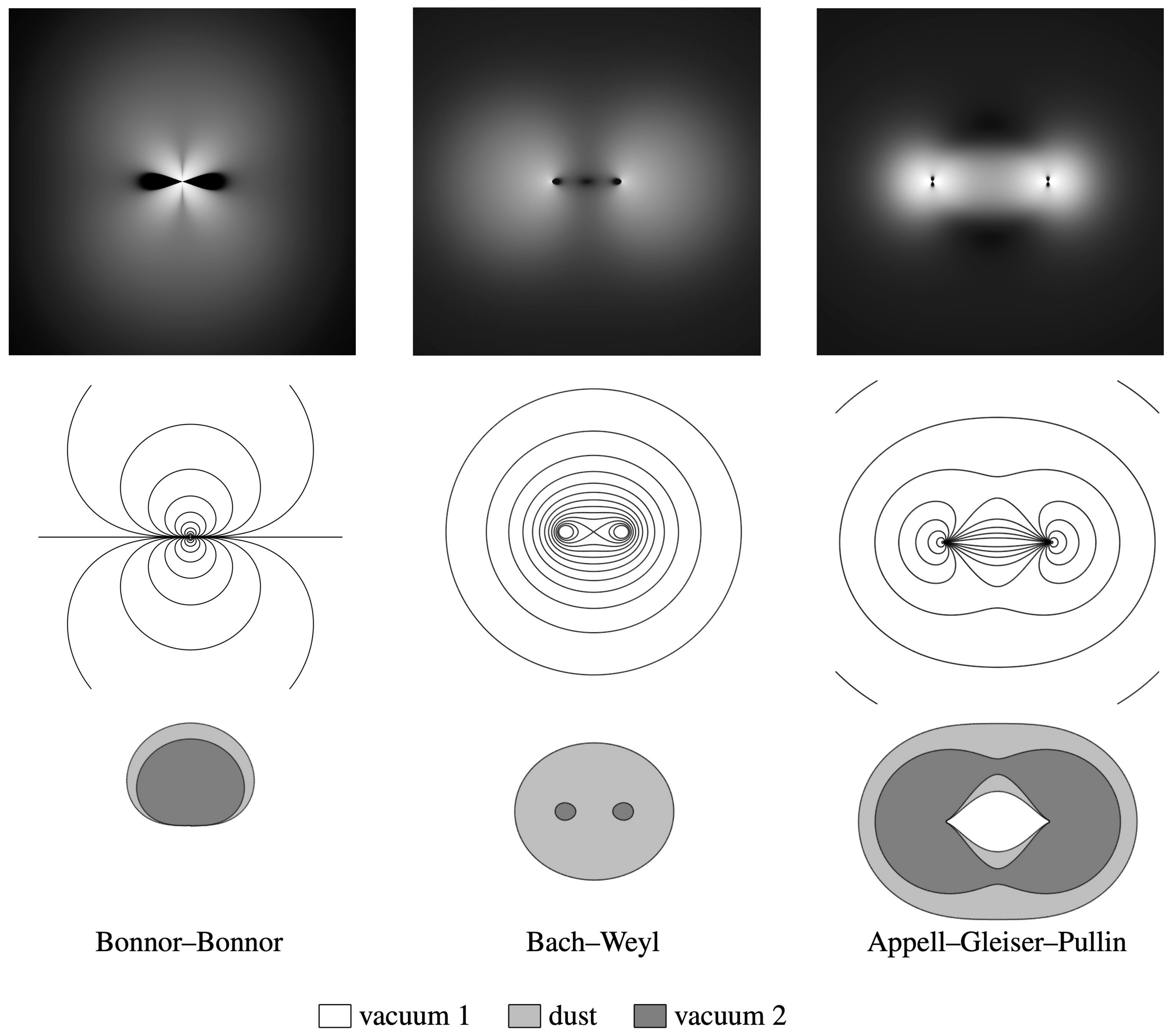
M.M. (2023)

Thus, our solution is still incomplete.

Meaning and perspectives

- Rotating Papapetrou vacua have no known physical interpretation other than being a zero-mass limit – Sackfield (1971), Bonnor (2005).
- Serious doubts persist as to whether a bounded rotating cloud of van Stockum’s dust can exist in nature for two reasons:
  - ▲ occurrence of closed time-like curves in rotationally interpreted metrics;
  - ▲ unavoidable negative-mass singularity – Caporali (1978), Frauendiener (1987), Gürlebeck (2009), Pfister (2010), Rowland (2015), Zingg et al. (2007).
- Yet Ilyas et al. (2017) proposed measurements to identify the Bonnor (1977) dust in the galaxy centre.
- The search for possible generalisations, especially ones leading to more realistic composite C^1-metrics, is under way.

Pictures



Line 1: dust densities. Line 2: possible boundaries. Line 3: example dust clouds.

The paper

For details see the paper

M.M., Matching van Stockum dust to Papapetrou vacuum, *J. Geom. Phys.* **190** (2023) 104878.

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Dedication

In commemoration of prof. Jan Horský (1940–2012)